# MATH 9: HOMEWORK WEEK 11

2020 DECEMBER 13

## 1. Homework

**1.** Out of the following relations, determine whether it's an equivalence relation; if it's an equivalence relation, describe its equivalence classes; if it's not an equivalence relation, explain why it isn't.

(a) On $\mathbb{R}^2 = \mathbb{R} \times \mathbb{R}$, the relation given by $(a_1, a_2) \sim (b_1, b_2)$ if $a_1 + a_2 = b_1 + b_2$

(b) On the set of all line segments in the plane, $l \sim m$ if the two segments form parallel sides of a parallelogram

(c) On the set of all subsets of $\mathbb{N}$, $A \sim B$ if the minimum element of $A$ is equal to the minimum element of $B$

(d) On the set $\mathbb{Z}^2$, $(a_1, a_2) \sim (b_1, b_2)$ if $a_1 b_2 = a_2 b_1$

**2.** Out of the following relations, determine whether it's an equivalence relation; if it's an equivalence relation, describe its equivalence classes; if it's not an equivalence relation, explain why it isn't.

(a) On the set $L$ of all leaves, given leaves $a$ and $b$, $a \sim b$ if $a$, $b$ are on the same plant

(b) Out of all locations on Earth, two locations are equivalent if the maximum height (azimuth angle) of the sun in the sky throughout the year is the same

(c) On the set of all classes in some school, two classes are equivalent if one is a prerequisite or corequisite for the other (you can consider a class to be a corequisite of itself)

(d) Out of all locations on Earth, two locations are equivalent if they are antipodes

**3.** Develop definitions for the special binary relations. The attributes *function, injective, surjective, bijective,* and *order* can all be defined as a subset of the nine main properties listed: { right-total, left-total, right-definite, left-definite, reflexive, irreflexive, symmetric, antisymmetric, transitive }. Figure out which subset of this set of nine properties is a reasonable definition for each of the following attributes, and explain your reasoning.

(a) Function. A relation is a function if...

(b) Injective. A relation is injective if...

(c) Surjective. A relation is surjective if...

(d) Bijective. A relation is bijective if...

(e) Order. A relation is an order relation if...

**4.** For the Square equivalence relation (Example 4) written on the main sheet, explain why each of the given properties is true or false. In other words, explain why this relation is left-total and right-definite, and also why it is not right-total, left-definite, reflexive, irreflexive, symmetric, anti-symmetric, or transitive.

**5.** For the relation on $\mathbb{R}$ given by $x \sim y$ if $x - y \in \mathbb{Z}$, describe the equivalence classes of this relation. Find, using your opinion/judgment, the nicest way to plot or depict these classes in a diagram. There is a familiar mathematical word/concept that nicely describes this set of equivalence classes, try to guess which one it is.

**6.** Prove the Finite Integral Domain theorem. I.e., prove that an equivalence class mod $n$ is invertible if and only if it is not a zero-divisor.

**7.** Give an example of a binary relation that is antisymmetric and reflexive, or prove that it's impossible.

**8.** Given two nonconcentric circles,

(a) Determine, with proof, the locus of points whose power to each circle is the same. (Recall that the power of a point to a circle is $d^2 - r^2$, where $d$ is the distance from the point to the circle's center, and $r$ is the circle's radius).

*(b) Provide a straightedge-compass construction of this locus.