MATH 9: GEOMETRY REVIEW

DECEMBER 6, 2020

1. Classwork

Today we'll be using the whiteboard to try various drawing exercises and such.
Here are some exercises to use

1. Calculate $\binom{5}{3}$

2. Calculate $1+3+9+27+81+243$ (geometric series). Then calculate $1+\frac{1}{3}+\frac{1}{9}+\frac{1}{27}+\frac{1}{81}+\frac{1}{243}$.

3. Prove that a rhombus is a parallelogram. (A rhombus is a quadrilateral whose sides are all congruent)

4. Prove that, if two circles intersect at two points, then the line segment connecting their intersection points is perpendicular to the line segment connecting their centers.

5. Given a line segment, explain how to perform a straightedge-compass construction of a square on that segment (i.e. the given segment is one of the sides of the square).

6. Solve $x^2+\frac{1}{x^2}=\frac{17}{4}$

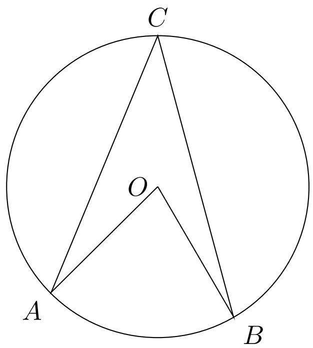
7. A pair of primes is said to be twin primes if there is only one number between them - for example, 5 and 7, or 11 and 13. Determine the first few twin prime pairs.

8. How many numbers are there from 1000 to 9999 whose digits contain at least one 1? What is the sum of all these numbers?

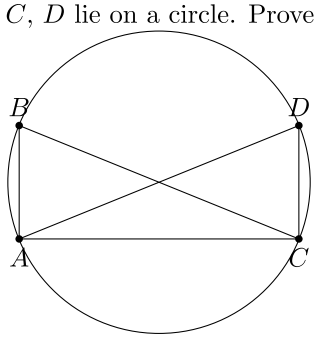
2. Problems

1. Let A, B, C be on a circle centered at O such that $\angle AOB \cong \angle BOC \cong \angle COA$. Prove that $\triangle ABC$ is an equilateral triangle.

2. Prove the Inscribed Angle Theorem for the case where the center of the circle is inside the angle. I.e., given an angle $\angle AXB$ where A, B, and C lie on a circle centered at O, such that O is inside $\angle ACB$, prove that $\angle ACB = \frac{1}{2}\angle AOB$.



3. Let $\triangle ABC$ and $\triangle ADC$ be right triangles (with hypotenuses BC and AD respectively) such that A, B, C, D lie on a circle. Prove that $ABCD$ is a rectangle.



4. Prove the triangle angle sum theorem, i.e. given any triangle, the sum of its angles is $180°$.

5. Given four points A, B, C, and D, construct (with straightedge and compass) an isosceles triangle such that A and B are on different legs and C and D are on the base. You may assume that the four points are chosen so that this construction is possible.

6. Given a trapezoid whose base angles are both $30°$,, let the ratio of the bigger base to the smaller base be r, for some $r > 1$. Let the trapezoid's height (the distance between its bases) be h. Determine, with proof, the length of the trapezoid's longer base, in terms of r and h.

7. Given four points, construct (with straightedge and compass) a rectangle such that each point lies on a different side of the rectangle. You may assume that the four points are chosen so that this construction is possible.

8. Given a regular n-gon, let A, B, C, D be adjacent vertices of the n-gon, in that respective order. Prove that $\frac{1}{AB} = \frac{1}{AC} + \frac{1}{AD}$.

9. Construct (with straightedge and compass) a square over a circle such that the square cuts off four congruent arcs of the circle (i.e. there are four congruent arcs of the circle inside the square), whose total angle sum is $120°$.