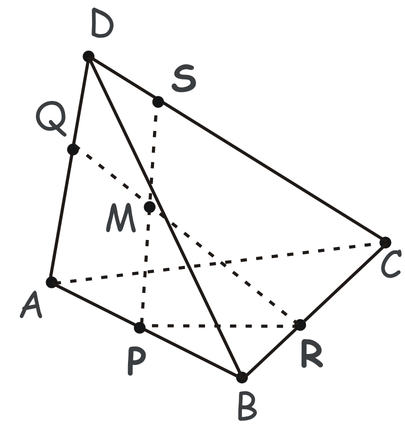
Homework due November 15, 2020.

Geometry.

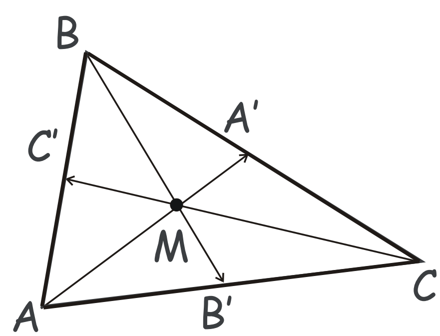
Review the classwork handout. Solve the unsolved problems from previous homeworks. Solve the following problems from the previous homework using the method of point masses and the Law of Lever (review the solutions for problems that were discussed in class).

Problems.

1. Each vertex of the tetrahedron $ABCD$ is connected with the centroid of the opposite face (the crossing point of its medians). Prove that all four of these segments, as well as the segments connecting the midpoints of the opposite edges (opposite edges have no common points; there are three pairs of opposite edges in a tetrahedron, and therefore three such segments) – seven segments in total, have common crossing point (are concurrent).



2. In a quadrilateral $ABCD$, E and F are the mid-points of its diagonals, while O is the point where the midlines (segments conneting the midpoints of the opposite sides) cross. Prove that E, F, and O are collinear (belong to the same line).
3. In a triangle ABC, Cevian segments AA', BB' and CC' are concurrent and cross at a point M (point C' is on the side AB, point B' is on the side AC, and point A' is on the side BC). Given the ratios $\frac{AC'}{C'B} = p$ and $\frac{AB'}{B'C} = q$, find the ratio $\frac{AM}{MA'}$ (express it through p and q).



4. In a parallelogram $ABCD$, a line passing through vertex D passes through a point E on the side AB, such that $|AE|$ is $1/n$-th of $|AB|$, n is an integer. At what distance from A, relative to the length, $|AC|$, of the diagonal AC it meets this diagonal?

Algebra.

Review the previous classwork handout. Solve the remaining problems from the previous homework assignments and classwork exercises. Try solving the following problems.

1. Using Euclid's algorithm, provide the continued fraction representation for the following numbers. Using the calculator, compare the values obtained by truncating the continued fraction at 1st, 2nd, 3rd, … level with the value of the number itself (in decimal representation).
   a. $\frac{1351}{780}$
   b. $\frac{25344}{8069}$
   c. $\frac{29376}{9347}$
   d. $\frac{6732}{1785}$
   e. $\frac{2187}{2048}$
   f. $\frac{3125}{2401}$
2. Is there a number, x, represented by the following infinite continued fraction? If so, find it.
   a. $x = 5 - \frac{6}{5-\frac{6}{5-\frac{6}{5-\cdots}}}$
   b. $x = 2 - \frac{1}{2-\frac{1}{2-\frac{1}{2-\cdots}}}$
   c. $x = 1 - \frac{6}{1-\frac{6}{1-\frac{6}{1-\cdots}}}$
3. Write the first few terms in the following sequence ($n \geq 1$),

$$n\ fractions \left\{ \frac{1}{1+\frac{1}{1+\frac{1}{1+\cdots}}} \atop \ldots + \frac{1}{1+x} \right. = ?$$

   a. Try guessing the general formula of this fraction for any n.

   b. Using mathematical induction, try proving the formula you guessed.

4. Can you prove that,

   a.

   $$\frac{3+\sqrt{17}}{2} = 3 + \frac{2}{3+\frac{2}{3+\frac{2}{3+\cdots}}}?$$

   b. $1 = 3 - \frac{2}{3-\frac{2}{3-\frac{2}{3-\cdots}}}?$

   c.

   $$\frac{4}{2+\frac{4}{2+\frac{4}{2+\cdots}}} = 1 + \frac{1}{4+\frac{1}{4+\frac{1}{4+\cdots}}}?$$