

Review of selected homework problem.

Problem 2. Solve the following equation:

$$\text{a. } \frac{x-a}{x-b} + \frac{x-b}{x-a} = 2.5$$

Solution.

$$\begin{aligned} \left(\frac{x-a}{x-b} + \frac{x-b}{x-a} = 2.5 \right) &\Leftrightarrow [(x-a)^2 + (x-b)^2 = 2.5(x-a)(x-b)] \wedge (x \neq a) \wedge \\ &(x \neq b) \Leftrightarrow [(2x^2 - 2(a+b)x + a^2 + b^2 = 2.5(x^2 - (a+b)x + ab))] \wedge \\ &(x \neq a) \wedge (x \neq b) \Leftrightarrow [(0.5x^2 - 0.5(a+b)x - a^2 + 2.5ab - b^2 = 0) \wedge \\ &(x \neq a) \wedge (x \neq b)] \Leftrightarrow [(x^2 - (a+b)x - 2(a-b)^2 + ab = 0) \wedge (x \neq a) \wedge \\ &(x \neq b)] \Leftrightarrow \left[\left(x = \frac{a+b}{2} \pm \sqrt{\frac{1}{4}(a+b)^2 + 2(a-b)^2 - ab} = 0 \right) \wedge (x \neq a) \wedge \right. \\ &\left. (x \neq b) \right] \Leftrightarrow \left[\left(x = \frac{a+b}{2} \pm \sqrt{\frac{9a^2 - 18ab + 9b^2}{4}} = \frac{a+b}{2} \pm 3\frac{a-b}{2} \right) \wedge (x \neq a) \wedge (x \neq b) \right] \\ &\Leftrightarrow [(x = 2a - b) \vee (x = 2b - a) \wedge (a \neq b)] \end{aligned}$$