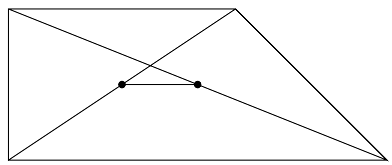
MATH 9: HOMEWORK 2

SEPTEMBER 27, 2020

1. Geometry Problems

1. Prove that the line segment connecting the midpoints of the diagonals of a trapezoid is parallel to the bases of the trapezoid.



2. Let A, B, C be on a circle centered at O such that $\angle AOB \cong \angle BOC \cong \angle COA$. Prove that $\triangle ABC$ is an equilateral triangle.

3. Given a line segment of length 1 and some positive integer a, use straightedge and compass to construct a line segment of length:

(a) a
(b) $\frac{1}{a}$
(c) $\sqrt{a}$

4. Prove that two distinct circles cannot intersect at three distinct points.

5. Is it possible to make a single slice in a tetrahedral block of cheese so that one of the faces of the remaining block (without the slice) is a rectangle?

2. Logic and Algebra Problems

1. Given logical statements m, p, q, let a denote the combined statement $(m \wedge p) \vee (\neg m \wedge q)$. In other words, $a \leftrightarrow ((m \wedge p) \vee (\neg m \wedge q))$. Prove the following:

(a) If m is true, then $a \leftrightarrow p$
(b) If m is false, then $a \leftrightarrow q$

2. Simplify $a \leftrightarrow (b \leftrightarrow (c \leftrightarrow (a \leftrightarrow (b \leftrightarrow c))))$

3. A function f on the integers (or in general) is said to be *involutive* if $f(f(n)) = n$ for all n. Can you find an involutive function that satisfies $f(0) = 1$?

4. Suppose you have a function (defined on the real numbers) that satisfies the following property for any pair of numbers x, y: $f\left(xf(y)\right) = yf(x) + 1 - y$. Is it possible to determine any values of f for certain? Try plugging in some numbers and/or variables and see what happens.

3. Additional Problem

1. How many five letter "words" with distinct letters are there such that the letters are in alphabetical order? For the purposes of this problem, define a "word" as any sequence of letters from the alphabet a through z; "distinct letters" means all five letters should be different.