

Homework for March 28, 2021.

Algebra.

Review the classwork handout. Try solving the following problems.
Remember: you do not necessarily need to solve all problems, just solve as many as you can within the time you can dedicate to Math 9 homework.

1. From the picture, find in which interval(s) the function $y = f(x)$

a. is monotonic

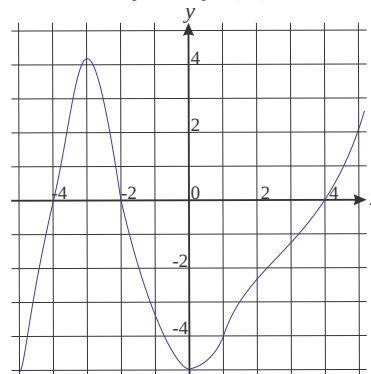
b. has the same sign

2. Find all possible values of a such that equation $x^2 + ax + 9 = 0$ has two different roots, both of which are less than -1 .

3. Draw graphs of the following functions

a. $y = \left| \frac{1}{x-2} + 1 \right|$

b. $y = \frac{1}{|x|-2} + 1$



4. Solve the following equations

a. (Skanavi 7.141) $3 \cdot 4^x + \frac{1}{3} \cdot 9^{x+2} = 6 \cdot 4^x - \frac{1}{2} \cdot 9^{x+1}$

b. (Skanavi 7.143) $\sqrt{\log_x \sqrt{x}} = -\log_x 5$

c. (Skanavi 7.153) $\frac{\log_2(9-2^x)}{3-x} = 1$

d. (Skanavi 7.160) $\log_a x + \log_{a^2} x + \log_{a^3} x = 11$

e. (Skanavi 7.184) $2^{x-1} + 2^{x-4} + 2^{x-2} = 6.5 + 3.25 + 1.625 + \dots$

f. (Skanavi 7.190) $9^x + 6^x = 2^{2x+1}$

g. (Skanavi 7.197) $4^{\log x+1} - 6^{\log x} - 2 \cdot 3^{\log x^2+2} = 0$

h. (Skanavi 7.299) $(x^2 - x - 1)^{x^2-1} = 1$

i. (Skanavi 7.304) find integer root: $\log_{\sqrt{x}}(x+12) = 8 \log_{x+12} x$

j. (Skanavi 7.308) $\log_{x+3}(3 - \sqrt{1-2x+x^2}) = \frac{1}{2}$

5. (Skanavi 7.277) Equation $4^x + 10^x = 25^x$ has a single root. Find this root. Is it positive or negative? Is it larger or less than 1?

6. (Skanavi 7.280) Show that:

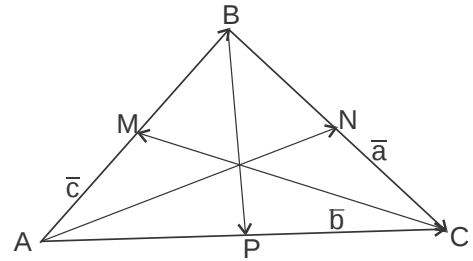
$$\log_3 2 \cdot \log_4 3 \cdot \log_5 4 \cdot \log_6 5 \cdot \log_7 6 \cdot \log_8 7 = \frac{1}{3}$$

Geometry.

Please, complete the previous homework assignments from this year. Review the classwork handout on vectors. Solve the following problems.

Problems.

- In a triangle ABC , vectors $\overrightarrow{AB}$, $\overrightarrow{AC}$ and $\overrightarrow{BC}$ ($\mathbf{c}$, $\mathbf{b}$ and $\mathbf{a}$) are the sides. $\overrightarrow{AN}$, $\overrightarrow{CM}$ and $\overrightarrow{BP}$ are the medians.
 - Express vectors $\overrightarrow{AN}$, $\overrightarrow{CM}$ and $\overrightarrow{BP}$ through vectors $\mathbf{c}$, $\mathbf{b}$ and $\mathbf{a}$.
 - Find the sum of vectors $\overrightarrow{AN}$, $\overrightarrow{CM}$ and $\overrightarrow{BP}$.
- Solve the same problem for bisectors $\overrightarrow{AN}$, $\overrightarrow{CM}$ and $\overrightarrow{BP}$ in a triangle ABC .
- Coxeter, Greitzer, problem #9 to Sec. 2.1 (p. 31): How far away is the horizon as seen from the top of a mountain 1 mile high? (Assume the Earth to be a sphere of diameter 7920 miles.)
- In a rectangle $ABCD$, A_1 , B_1 , C_1 and D_1 are the mid-points of sides AB , CD , BC and DA , respectively. M is the crossing point of the segments A_1B_1 , and C_1D_1 , connecting two pairs of midpoints.
 - Express vector $\overrightarrow{A_1M}$ through $\overrightarrow{AB}$, $\overrightarrow{BC}$ and $\overrightarrow{CD}$.
 - Prove that M is the mid-point of segments, A_1B_1 and C_1D_1 , i.e. $|A_1M| = |MB_1|$ and $|C_1M| = |MD_1|$.
- In a parallelogram $ABCD$, find $\overrightarrow{AB} + \overrightarrow{BD} - 2\overrightarrow{AD}$.
- M is a crossing point of the medians in a triangle ABC . Prove that $\overrightarrow{AM} = \frac{1}{3}(\overrightarrow{AB} + \overrightarrow{AC})$.
- For three points, $A(-1,3)$, $B(2,-5)$ and $C(3,4)$, find the (coordinates of) following vectors,
 - $\overrightarrow{AB} - \overrightarrow{BC}$
 - $\overrightarrow{AB} + \overrightarrow{CB} + \overrightarrow{AC}$



- c. $\overrightarrow{AB} + \frac{1}{2}\overrightarrow{BC} + \frac{1}{3}\overrightarrow{CA}$
8. For two triangles, ABC and $A_1B_1C_1$, $\overrightarrow{AA_1} + \overrightarrow{BB_1} + \overrightarrow{CC_1} = 0$. Prove that medians of these two triangles cross at the same point M .

Trigonometry exercise.

Review the trigonometry classwork handout. Solve the following problems. Some problems are repeated from previous trigonometry homeworks – skip those that you have already solved.

- Find all x for which,
 - $\sin x \cos x = \frac{\sqrt{2}}{2}$
 - $\sin x \cos x = \frac{1}{2}$
 - $\sin x \cos x = \frac{\sqrt{3}}{4}$
- Find the sum of the following series,

$$S = \cos x + \cos 3x + \cos 5x + \cos 7x + \cdots + \cos 2021x$$

(hint: multiply the sum by $2 \sin x$)

- Calculate:
 - $\cos 75^\circ + \cos 15^\circ =$
 - $\cos \frac{\pi}{12} - \cos \frac{5\pi}{12} =$
- Let A, B and C be angles of a triangle. Prove that

$$\tan \frac{A}{2} \tan \frac{B}{2} + \tan \frac{B}{2} \tan \frac{C}{2} + \tan \frac{C}{2} \tan \frac{A}{2} = 1$$

- Prove the following equalities:

$$\text{a. } \frac{1}{\sin \alpha} + \frac{1}{\tan \alpha} = \cot \frac{\alpha}{2}$$

$$\text{b. } \sin^2 \left(\frac{7\pi}{8} - 2\alpha \right) - \sin^2 \left(\frac{9\pi}{8} - 2\alpha \right) = \frac{\sin 4\alpha}{\sqrt{2}}$$

$$\text{c. } (\cos \alpha - \cos \beta)^2 + (\sin \alpha - \sin \beta)^2 = 4 \sin^2 \frac{\alpha - \beta}{2}$$

$$\text{d. } \frac{\cot^2 2\alpha - 1}{2 \cot 2\alpha} - \cos 8\alpha \cot 4\alpha = \sin 8\alpha$$

- e. $\sin^6 \alpha + \cos^6 \alpha + 3 \sin^2 \alpha \cos^2 \alpha = 1$
- f. $\frac{\sin 6\alpha + \sin 7\alpha + \sin 8\alpha + \sin 9\alpha}{\cos 6\alpha + \cos 7\alpha + \cos 8\alpha + \cos 9\alpha} = \tan \frac{15\alpha}{2}$
- g. $\sin^6 \alpha + \cos^6 \alpha = \frac{5 + 3 \cos 4\alpha}{8}$
- h. $16 \sin^5 \alpha - 20 \sin^3 \alpha + 5 \sin \alpha = \sin 5\alpha$
- i. $\frac{\cos 64^\circ \cos 4^\circ - \cos 86^\circ \cos 26^\circ}{\cos 71^\circ \cos 41^\circ - \cos 49^\circ \cos 19^\circ}$
- j. $\sin 20^\circ \sin 40^\circ \sin 60^\circ \sin 80^\circ = \frac{3}{16}$
- k. $\frac{1}{\sin 10^\circ} - \frac{\sqrt{3}}{\cos 10^\circ} = 4$

6. Simplify the following expressions:

- a. $\sin^2 \left(\frac{\alpha}{2} + 2\beta \right) - \sin^2 \left(\frac{\alpha}{2} - 2\beta \right)$
- b. $2 \cos^2 3\alpha + \sqrt{3} \sin 6\alpha - 1$
- c. $\cos^4 2\alpha - 6 \cos^2 2\alpha \sin^2 2\alpha + \sin^4 2\alpha$
- d. $\sin^2(135^\circ - 2\alpha) - \sin^2(210^\circ - 2\alpha) - \sin^2 195^\circ \cos(165^\circ - 4\alpha)$
- e. $\frac{\cos 2\alpha - \cos 6\alpha + \cos 10\alpha - \cos 14\alpha}{\sin 2\alpha + \sin 6\alpha + \sin 10\alpha + \sin 14\alpha}$

7. Solve the following equations and inequalities:

- a. $\cos^2 \pi x + 4 \sin \pi x + 4 = 0$
- b. $\sin x + \sin 2x + \sin 3x = \cos x + \cos 2x + \cos 3x$
- c. $\cos 3x - \sin x = \sqrt{3}(\cos x - \sin 3x)$
- d. $\sin^2 x - 2 \sin x \cos x = 3 \cos^2 x$
- e. $\sin 6x + 2 = 2 \cos 4x$
- f. $\cot x - \tan x = \sin x + \cos x$
- g. $\sin x \geq \pi/2$
- h. $\sin x \leq \cos x$