MATH 8
PRESIDENTIAL PARDON TEST
DUE: FEBRUARY 20, 2021

This test is designed for those of you who either did not submit many of the previous homework assignments or for those of you who got a score of 1 on them. If you are unsure whether you have to do the test, please contact me at antonenko@schoolnova.org.

In order to pass this course and transfer to the next level of Math at SchoolNova, students must submit at least 80% of homework assignments. We are aware that some of you have fallen behind on it – completing this test would allow you to get back on track. Doing this will excuse you from having to submit all previous homework assignments (up to Handout 15) that you have not yet submitted or did not do well on to be able to transfer to the next level of Math.

1. Combinatorics

1. Explain in your own words what you have learned or reviewed about combinatorics in this class.

2. Write out the formula for $\binom{n}{k}$. Explain each of the parts of the formula and why they are there.

3. How many different sequences of letters could you make from the letters of the word "alfalfa"?

4. There are 8 people in a house. This is over capacity: the fire capacity of the house is 7. How many ways are there to kick one person out of the house?
In how many ways could you get the house down to capacity if the fire capacity were 6 instead of 7?

5. How many ways are there to choose a committee of three people from a group of six people? What if one of the committee members must be selected to be president?

6. How many 2-digit numbers are there where the digits are in decreasing order? For example, 54 and 73 and 86 and 50 count, but 12 and 46 and 68 and 55 do not count.

7. Two schools are hosting an exchange program. Each school will choose 5 students to travel to the other school. If one school has 40 students and the other has 50, how many possible ways are there to make the exchange?

8. Let p be prime.
(a) Show that each of the binomial coefficients $\binom{p}{k}$, $1 \leq k \leq p-1$, is divisible by p.
(b) Show that if a, b are integer, then $(a+b)^p - a^p - b^p$ is divisible by p.

2. Logic

9. Explain in your own words what you have learned about logic in this class.

10. Describe one point about logic you learned that you have some opinion about (interesting, cool, weird, unexpected, etc.).

11. Prove that $(A \implies B) \wedge (B \implies A)$ is equivalent to $A \leftrightarrow B$

12. A teacher tell the student "If you do not take the final exam, you get an F". Does it mean that
(a) If the student does take the final exam, he will not get an F
(b) If the student does not get an F, it means he must have taken the final exam.

13. Use the truth tables to prove *De Morgan's laws*

$$\neg(A \wedge B) \leftrightarrow (\neg A) \vee (\neg B)$$
$$\neg(A \vee B) \leftrightarrow (\neg A) \wedge (\neg B)$$

14. Suppose you have a square $ABCD$ with side length 1. For some point x on the square, let Ax be the distance from x to A, Bx be the distance from x to B, etc.
(a) Write out a logical statement that is true for all points on the square except the four vertices.

(b) Write out a logical statement that is true only for points on one side of the square.
(c) Write out a logical statement that is true only for points on two opposite sides of the square.

3. Geometry

15. Explain in your own words what you have learned about geometry so far in this class.

16. State one of the geometry theorems we have discussed that you found difficult or interesting. Write out why you think you find this theorem difficult or interesting.

17. Suppose you have a quadrilateral whose four sides are all congruent. Prove that the diagonals of this quadrilateral are perpendicular.

18. In an isosceles triangle, the angle bisector from the apex to the base is also the altitude from the vertex to the base. Is it also true for all isosceles triangles that the angle bisector from one of the base vertices is the same as the altitude from that vertex to the opposite side? Provide a proof or counterexample.

19. Suppose you have a triangle $\triangle ABC$. Let D be a point on $\overline{AB}$ such that $\overline{DC}$ is the altitude from C to AB. Prove that D is the midpoint of $\overline{AB}$ if and only if $\triangle ABC$ is isosceles with apex C.