MATH 8: HANDOUT 5
MORE COMBINATORICS

These are some additional combinatorics problems.

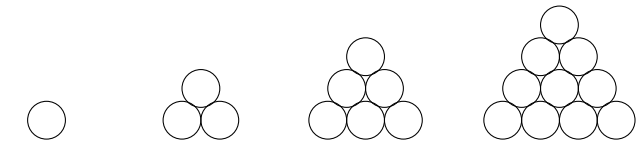
1. How many ways there are to group $2n$ people into n pairs? [Pairs are not numbered.]

2. How many different "words" can be formed by permuting letters of the word "letter"? of the word "Mississippi"? (by "word", we mean any sequence of letters, not necessarily meaningful").

3. How many ways there are to arrange 12 books on 2 bookshelves (top and bottom one)? The order on each bookshelf matters; there are no restrictions on how many of the 12 books are on top shelf.

4. How many different monomials in 3 variables x, y, z of total degree n are there? in 4 variables?

5. Let T_n be the number of circles in a triangular shape with n levels like the ones below (these are sometimes called *triangular* numbers):



$T_1 = 1$ $T_2 = 3$ $T_3 = 6$ $T_4 = 10$

(a) Note that $T_3 = T_2 + 3$, and $T_4 = T_3 + 4$. Is it true that in general, $T_n = T_{n-1} + n$? Why or why not?

(b) Look at Pascal triangle. Can you find these numbers there?

(c) Can you write a general formula for T_n?

***6.** What if instead of drawing circles on plane, we were arranging balls in a pyramid? Can you guess how many balls we would have in pyramid with 1 level; with 2, 3, 4 levels? Can you find these numbers in Pascal triangle?