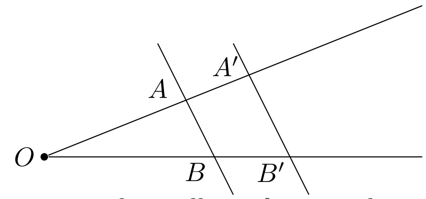
MATH 8: EUCLIDEAN GEOMETRY 6

FEB 21, 2021

Thales Theorem



Theorem 33. *Let points A', B' be on the sides of angle $\angle AOB$ as shown in the picture. Then lines AB and $A'B'$ are parallel if and only if*

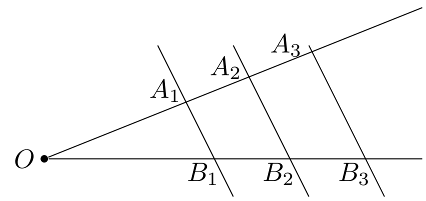
$$\frac{OA}{OB} = \frac{OA'}{OB'}$$

In this case, we also have $\frac{OA}{OB} = \frac{AA'}{BB'}$

We have already seen and proved a special case of this theorem when discussing the midline of a triangle.

The proof of this theorem is unexpectedly hard. In the case when $\frac{OA}{OA'}$ is a rational number, one can use arguments similar to those we did when talking about midline. The case of irrational numbers is harder yet. We skip the proof for now; it will be discussed in Math 9.

As an immediate corollary of this theorem, we get the following result.



Theorem 34. *Let points $A_1, \dots, A_n$ and $B_1, \dots B_n$ on the sides of an angle be chosen so that $A_1A_2 = A_2A_3 = \dots = A_{n-1}A_n$, and lines A_1B_1, A_2B_2, ... are parallel. Then $B_1B_2 = B_2B_3 = \dots = B_{n-1}B_n$.*

Proof of this theorem is left to you as exercise.

Similar triangles

Definition. Two triangles $\triangle ABC$, $\triangle A'B'C'$ are called *similar* if

$$\angle A \cong \angle A', \quad \angle B \cong \angle B', \quad \angle C \cong \angle C'$$

and the corresponding sides are proportional, i.e.

$$\frac{AB}{A'B'} = \frac{AC}{A'C'} = \frac{BC}{B'C'}$$

The common ratio $\frac{AB}{A'B'} = \frac{AC}{A'C'} = \frac{BC}{B'C'}$ is sometimes called the similarity coefficient.

There are some similarity tests:

Theorem 35 (AAA similarity test)**.** *If the corresponding angles of triangles $\triangle ABC$, $\triangle A'B'C'$ are equal:*

$$\angle A \cong \angle A', \quad \angle B \cong \angle B', \quad \angle C \cong \angle C'$$

then the triangles are similar.

Theorem 36 (SSS similarity test)**.** *If the corresponding sides of triangles $\triangle ABC$, $\triangle A'B'C'$ are proportional:*

$$\frac{AB}{A'B'} = \frac{AC}{A'C'} = \frac{BC}{B'C'}$$

then the triangles are similar.

Theorem 37 (SAS similarity test)**.** *If two pairs of corresponding sides of triangles $\triangle ABC$, $\triangle A'B'C'$ are proportional:*

$$\frac{AB}{A'B'} = \frac{AC}{A'C'}$$

and $\angle A \cong \angle A'$ then the triangles are similar.

Proofs of all of these tests can be obtained from Thales theorem.

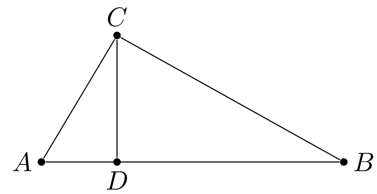
HOMEWORK

1. Prove Theorem 34 (using Thales Theorem). Hint: let $k = \frac{OB_1}{OA_1}$; show that then $B_iB_{i+1} = kA_iA_{i+1}$.

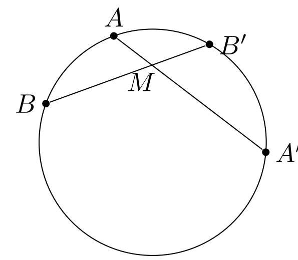
2. Using Theorem 34, describe how one can divide a given segment into 5 equal parts using ruler and compass.

3. Given segments of length a, b, c, construct a segment of length $\frac{ab}{c}$ using ruler and compass.

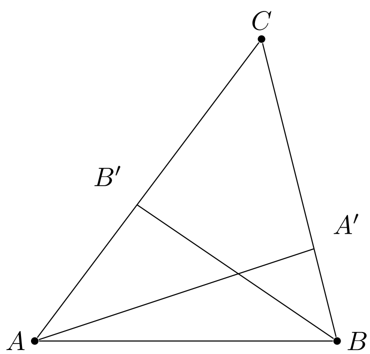
4. Let ABC be a right triangle, $\angle C = 90°$, and let CD be the altitude. Prove that triangles $\triangle ACD$, $\triangle CBD$ are similar. Deduce from this that $CD^2 = AD \cdot DB$.



5. Let M be a point inside a circle and let AA', BB' be two chords through M. Show that then $AM \cdot MA' = BM \cdot MB'$. [Hint: use inscribed angle theorem to show that triangles $\triangle AMB$, $\triangle B'MA'$ are similar.]



6. Let AA', BB' be altitudes in the acute triangle $\triangle ABC$.



(a) Show that points A', B' are on a circle with diameter AB.
(b) Show that $\angle AA'B' = \angle ABB'$, $\angle A'B'B = \angle A'AB$
(c) Show that triangle $\triangle ABC$ is similar to triangle $\triangle A'B'C$.