

**MATH 8**  
**ASSIGNMENT 8: CONDITIONALS**

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CONDITIONAL

In addition to all previous logic operations, there is one more which we have not yet fully discussed: implication, also known as conditional and denoted by  $A \implies B$  (reads A implies B, or “If A, then B”). It is defined by the following truth table:

$A$	$B$	$A \implies B$
T	T	T
T	F	F
F	T	T
F	F	T

Note that in particular, in all situations where  $A$  is false,  $A \implies B$  is automatically true. E.g., a statement “if  $2 \times 2 = 5$ , then...” is automatically true, no matter what conclusion one puts in place of dots.

Another logic operation is called equivalence and defined as  $(A \iff B)$  is true if  $A, B$  have the same value (both true or both false).

One can easily see that  $(A \iff B)$  is equivalent to  $(A \implies B) \text{ AND } (B \implies A)$ .

Also, implication is a logical relationship - it doesn't necessarily mean that  $A$  is the reason  $B$  is true. For example, you can say “if it is raining, then it is cloudy”, written as  $(\textit{raining}) \implies (\textit{cloudy})$ , and you can take a moment to think about why this makes sense.

PROBLEMS

1. Show that  $A \implies B$  is not equivalent to  $B \implies A$ ; one of them can be true while the other is false.
2. Prove the contrapositive law:  $A \implies B$  is equivalent to  $(\neg B) \implies (\neg A)$
3. Show that  $(A \implies B)$  is equivalent to  $B \vee \neg A$ . Can you rewrite  $\neg(A \implies B)$  without using implication operation?
4. Consider the following statement (from a parent to his son):  
“If you do not clean your room, you can't go to the movies”  
Is it the same as:
  - (a) Clean your room, or you can't go to the movies
  - (b) You must clean your room to go to the movies
  - (c) If you clean your room, you can go to the movies
5. English language (and in particular, mathematical English) has a number of ways to say the same thing. Can you rewrite each of the verbal statements below using basic logic operation (including implications), and variables  
 $A$ : you get grade A for the class  
 $B$ : you get score of 90 or above on the final exam  
(As you will realize, many of these statements are in fact equivalent)
  - (a) To get A for the class, it is required that you get 90 or higher on the final exam
  - (b) To get A for the class, it is sufficient that you get 90 or higher on the final exam
  - (c) You can't get A for the class unless you got 90 or above on the final exam
  - (d) To get A for the class, it is necessary and sufficient that you get 90 or higher on the final exam
6. Show that in all situations where  $A$  is true and  $A \implies B$  is true,  $B$  must also be true. [This simple rule has a name: it is called *Modus Ponens*.]
7. Show that if  $A \implies B$  is true, and  $B$  is false, then  $A$  must be false. [This is called *Modus Tollens*.]
8. Use truth tables to show that if  $A \implies B$  is true, and  $B \implies C$  is true, then  $A \implies C$  is also true [This would require a truth table with 8 rows; we will discuss less time-consuming ways of argument next time.]
- \*9. (a) Show that  $(A \implies B) \implies C$  is not equivalent to  $A \implies (B \implies C)$ .

- (b) Is there any logical relation you could put in place of the star  $\star$  in order to make this true?  
 $((A \implies B) \implies C) = (A \implies (B \star C))$
- (c) Is it true that  $(A \iff B) \iff C$  is equivalent to  $A \iff (B \iff C)$ ?