MATH 8: GEOMETRY 1

JANUARY 10, 2021

1. Homework

1. It is important that you know some geometry notation.
(a) What does the symbol $\parallel$ mean? How do you pronounce it? How would you read "$a \parallel b$"?
(b) What does the symbol $\perp$ mean? How would you say "$a \perp b$"?
(c) Suppose you have two points X and Y. What is the difference between $\overline{XY}$, $\overleftrightarrow{XY}$, $\overrightarrow{XY}$? What are each of these things called?
(d) Given three points E, F, G, what does $EF + FG$ mean?
(e) Given four points A, B, C, D, what does $m\angle ADC + m\angle BDC$ mean? If I tell you $m\angle ADC + m\angle BDC = 180°$, does that tell you any information about $m\angle ADC$ or $m\angle BDC$?
(f) What does the symbol $\triangle$ mean? For example, if A and B and C are points, what is $\triangle ABC$?

2. Write out a list of geometric concepts that are given in section 1 of the coursework. (Section 1 is the "Basic Objects" section.)

3. (a) What is a proof? Give an example.
(b) What is an axiom? Give an example.
(c) Consider the mathematics of integer arithmetic, where you add and multiply integers. Write out some axioms for integer arithmetic that you believe cover a few of its basic ideas.
(d) Use your axioms from the previous part of this problem to prove that the sum of two positive integers is never equal to either of the two integers.

4. (a) In your own words, what is an abstract concept?
(b) What is the difference between a specific point and the concept of points in general? Explain this in whatever way you understand it.

5. (a) Suppose you have lines a, b and a point X such that X is on a but not on b. Is it possible for a, b to be parallel?
(b) Suppose, in another scenario, you have lines a, b with $a \parallel b$, and a line c. Is it possible that $a \perp c$ and $b \parallel c$?
(c) Suppose now you have lines a, c with $a \perp c$, and another line b. Is it possible that $a \perp b$ and $c \parallel b$?

6. The following logic statements come in equivalent pairs. For each statement on the list, there is exactly one other statement that is equivalent to it. Sort these statements into their equivalent pairs, with an explanation of why the pairs you chose are equivalent.
(a) $\forall b(\exists a(b \parallel a))$
(b) Given any two distinct points, there is at least one line through both those points
(c) $\forall x(\forall y(x \parallel y))$
(d) $\neg\exists v(\exists u(\forall f(f = v \lor f = u)))$
(e) $\forall a(\forall b(a \parallel b))$
(f) If two lines are parallel and another line intersects one, then it intersects the other as well
(g) $\forall c(\forall y(\exists r(r \neq c \land r \neq y)))$
(h) $\neg\exists b(\neg\exists a(a \parallel b))$
(i) If two lines are parallel to the same line, then they are parallel to each other
(j) $\forall a(\forall b(a \neq b \implies \exists c(a \in c \land b \in c)))$ (although I did not explicitly mention which of these variables represent points and which represent lines, you can make your best guess)

7. Recall the following deduction rule: if $a \implies b$ is true and a is true, then b is true. Now, if it happens that a is an axiom, then you can deduce b just from $a \implies b$ because axioms are assumed to always be true. Write out a logical statement that is equivalent to the statement of theorem 1. Then, write out a logical statement that can be used to deduce theorem 1 (assuming that the axioms are true, and using the deduction rule explained above).

8. Suppose some object X is both a point and a line. Does this lead to any problems? Can you deduce a contradiction? Study section 1 ("basic objects") thoroughly, the information you need is in there. [Hint: how many points are on a line?]

9. In this problem, you will make diagrams. Part of the purpose of this exercise is so that, when you think about geometry, the pictures in your notes or in your mind aren't all just the diagrams I draw out for you in class or on classwork sheets. You have to be able to draw or visualize configurations of lines other than the way they're set up in axiom 4, for example.

(a) Given lines a, b, c, is it possible that $a \parallel b$ and $\neg(b \parallel c)$ but $a \parallel c$? Draw a diagram and then explain your reasoning on how to answer this question. ("explain" means, of course, in writing.)

(b) Prove theorem 2. Draw a diagram and then explain your proof.

(c) Prove theorem 6 by contradiction. Draw a diagram that helps you explain your proof.

(d) Suppose we have parallel lines l, m. Let A, B, C be points on l, with B between A, C. Let X, Y, Z be points on m, with Y between X, Z. Is it possible for lines $\overleftrightarrow{AX}$, $\overleftrightarrow{BY}$, $\overleftrightarrow{CZ}$ to all intersect at one point? Draw a diagram of what this might look like.

(e) Consider the diagram you drew in the previous part, with the lines l, m and the six points, and the three cross-lines that intersect at a point. Now consider the lines $\overleftrightarrow{AZ}$, $\overleftrightarrow{CX}$. Do these two lines intersect at a point on $\overleftrightarrow{BY}$? Draw a diagram where this *is* the case, and then draw a second diagram where this *is not* the case.

(f) Draw a rectangle that's not a square, and draw it so that one of the bases is horizontal. Then draw one of the recangle's diagonals. Notice that, of the two right angles formed at the rectangle's base, the rectangle's diagonal splits one of those angles into two smaller angles. Which of the two angles is bigger - the one below the diagonal, or the one above the diagonal? Draw a second rectangle where the opposite relation holds true (for example, if the lower angle was bigger in your first rectangle, draw a second rectangle where the lower angle split by the diagonal is smaller).