

MATH 8
WINTER CHALLENGE PROBLEMS
DECEMBER-JANUARY 2020-21

1. Pirate Deals

Five pirates have a stack of treasure: 100 gold coins. The captain proposes a deal on how to split it. The pirates (captain included) vote on the captain's proposal. If a majority disapprove, mutiny ensues and captainship is passed to the next living pirate in command.

The pirates have a strict hierarchy, from captain at the top to 5th rank at the bottom.

The proposal-voting procedure repeats until a majority agrees on a proposal (half counts as a majority if only an even number of pirates is alive).

Assuming the pirates are all perfectly logical and perfectly greedy, what will the captain propose?

- 2.** Suppose you have a line with equation $f(x) = ax + b$, and a quadratic function with equation $g(x) = cx^2$. The numbers a , b , c are all positive constants. Prove that there exists some positive r such that $g(r) > f(r)$, and some negative s such that $g(s) > f(s)$.