MATH 8
ASSIGNMENT 8: CONDITIONALS

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Conditional

In addition to all previous logic opeaations, there is one more which we have not yet fully discussed: implication, also known as conditional and denoted by $A \implies B$ (reads A implies B, or "If A, then B"). It is defined by the following truth table:

A	B	$A \implies B$
T	T	T
T	F	F
F	T	T
F	F	T

Note that in particular, in all situations where A is false, $A \implies B$ is automatically true. E.g., a statement "if $2 \times 2 = 5$, then..." is automatically true, no matter what conclusion one puts in place of dots.

Another logic operation is called equivalence and defined as $(A \iff B)$ is true if A, B have the same value (both true or both false).

One can easily see that $(A \iff B)$ is equivalent to $(A \implies B)\,\text{AND}(B \implies A)$.

Also, implication is a logical relationship - it doesn't necessarily mean that A is the reason B is true. For example, you can say "if it is raining, then it is cloudy", written as $(\textit{raining}) \implies (\textit{cloudy})$, and you can take a moment to think about why this makes sense.

Problems

1. Show that $A \implies B$ is not equivalent to $B \implies A$; one of them can be true while the other is false.

2. Prove the contrapositive law: $A \implies B$ is equivalent to $(\neg B) \implies (\neg A)$

3. Show that $(A \implies B)$ is equivalent to $B \vee \neg A$. Can you rewrite $\neg(A \implies B)$ without using implication operation?

4. Consider the following statement (from a parent to his son):

"If you do not clean your room, you can't go to the movies"

Is it the same as:

(a) Clean your room, or you can't go to the movies

(b) You must clean your room to go to the movies

(c) If you clean your room, you can go to the movies

5. English language (and in paricular, mathematical English) has a number of ways to say the same thing. Can you rewrite each of the verbal statements below using basic logic operation (including implications), and variables

A: you get score of 90 or above on the final exam

B: you get A grade for the class

(As you will realize, many of these statements are in fact equivalent)

(a) To get A for the class, it is required that you get 90 or higher on the midterm

(b) To get A for the class, it is sufficient that you get 90 or higher on the midterm

(c) You can't get A for the class unless you got 90 or above on the final exam

(d) To get A for the class, it is necessary and sufficient that you get 90 or higher on the midterm

6. Show that in all situations where A is true and $A \implies B$ is true, B must also be true. [This simple rule has a name: it is called *Modus Ponens*.]

7. Show that if $A \implies B$ is true, and B is false, then A must be false. [This is called *Modus Tollens*.]

***8.** (a) Show that $(A \implies B) \implies C$ is not equivalent to $A \implies (B \implies C)$.

(b) Is there any logical relation you could put in place of the star $\star$ in order to make this true?

$((A \implies B) \implies C) = (A \implies (B \star C))$

(c) Is it true that $(A \iff B) \iff C$ is equivalent to $A \iff (B \iff C)$?

***9.** Paper Folding:

Is it possible to fold a square origami paper in a way that you end up with a section whose area is exactly 1/3 the area of the paper? For which n is it possible to fold over a section whose area is 1/n that of the paper?

Origami rules:

The four sides of the square are 'lines' the four corners are 'points', and these are the only lines and points you start with. You may create a new point at the intersection of any two lines, and you may create new lines as:

-the line through two points

-the midline between two lines (the midline is the line of points equidistant from the two lines)

-the perpendicular to a line at a point (you can choose the point)

-the midline between two points

and, lastly, you may reflect any point or line through an existing line (ie you may fold the paper over a line and re-fold all creases to get their mirror images). These are the only allowed moves.

These rules are named after Huzita and Hatori, called the Huzita-Hatori axioms (except the last one), and include three more axioms which are uninteresting to this problem but useful in other adventures.

You can also play around with a piece of paper. The solution to this problem is actually possible to fold for smallish n, like $n < 12$, without too much difficulty (if you know what folds to make).