MATH 7: HANDOUT 23
TRIGONOMETRY 3: THE TRIGONOMETRIC CIRCLE

Radians

Until now, we have been measuring angles in degrees, which are defined by saying that a full turn corresponds to $360°$.

An alternative way to measure angles is by radians, which are defined in the following way: given an angle α, it's measure in radians is the ratio of an arc of circumference with angle α by the radius of the circumference.

For example, the angle $360°$ corresponds to a full circle. Since the perimeter of a circle is $2\pi R$, dividing by R gives:

$$360° \leftrightarrow 2\pi \text{ rad}.$$

In the same way, half a circle corresponds to an angle of π radians. By similar arguments, we can translate all the angles that appeared in our previous table:

Trigonometric Functions							
Function	Notation	Definition	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$
sine	$\sin(\alpha)$	$\frac{\text{opposite side}}{\text{hypotenuse}}$	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1
cosine	$\cos(\alpha)$	$\frac{\text{adjacent side}}{\text{hypotenuse}}$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0
tangent	$\tan(\alpha)$	$\frac{\text{opposite side}}{\text{adjacent side}}$	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	∞

.

Trigonometric Circle

A very useful tool in understanding the trigonometric functions is the *trigonometric circle* (see figure below): in order to find the sine and cosine of a positive angle α, we just have to "walk" around the circle a distance α, starting from the point $(1, 0)$ in anti clockwise direction. Then the coordinates of the point we arrive at are $(\cos\alpha, \sin\alpha)$. For α negative, we define the sine and cosine in the same way, but walking in the clockwise direction.

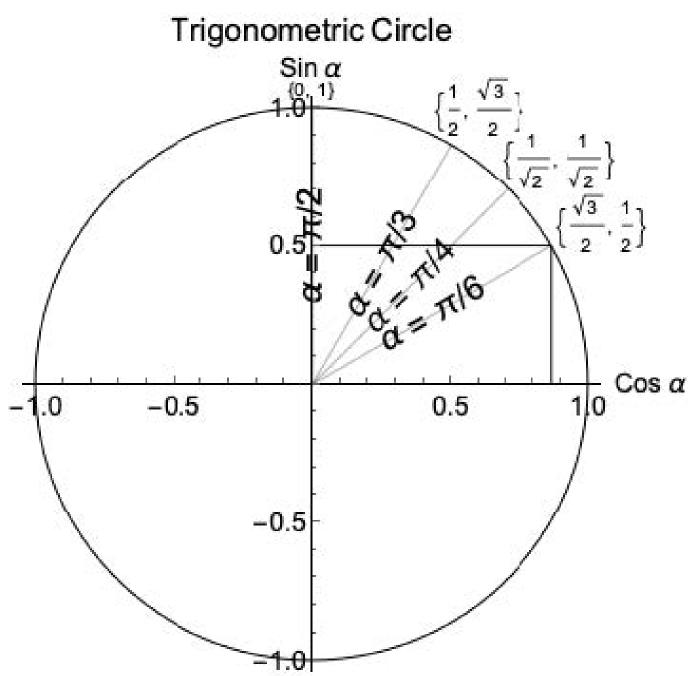

FIGURE 1. Trigonometric circle: in order to find the sine and cosine of angle α, we just have to "walk" around the circle a distance α, starting from the point $(1, 0)$. Then the coordinates of the point we arrive at are $(\cos\alpha, \sin\alpha)$.

HOMEWORK

1. Draw a large trigonometric circle. Then, remembering that 2π corresponds to a full circle, find the points corresponding to (write the corresponding letter on the correct point)

(a) π
(b) $\frac{3\pi}{2}$
(c) $\frac{3\pi}{4}$
(d) $-\frac{5\pi}{4}$
(e) 11π
(f) -3π
(g) $\frac{25\pi}{3}$
(h) $-\frac{19\pi}{6}$

2. Now use your trigonometric circle and figure 1 to complete this table:

Point	Sine	Cosine
(a)	0	-1
(b)		
(c)		
(d)		
(e)		
(f)		
(g)		
(h)		

.

3. Using the trigonometric circle, check where appropriate:

x	$\sin x \geq \sqrt{3}/2$	$1/2 < \sin x < \sqrt{3}/2$	$-\sqrt{2}/2 < \sin x \leq 1/2$	$\sin x \leq -\sqrt{2}/2$
$\pi/7$			✓	
$2\pi/7$				
$-3\pi/5$				
$5\pi/8$				
$25\pi/9$				

.

4. Using the trigonometric circle, show that $\cos x = \sin(x + \pi/2)$ for any angle x.

5. Find all real numbers x such that $(\sin x)^2 = 3/4$