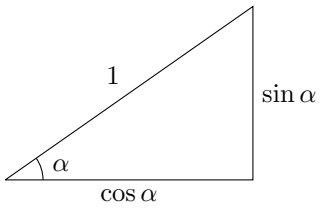
MATH 7: HANDOUT 21
TRIGONOMETRY 1: MAIN DEFINITIONS

BASIC TRIGONOMETRY

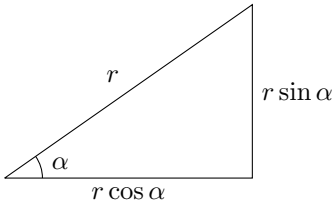
For any angle α, we define two numbers, $\sin\alpha$ (sine) and $\cos\alpha$ (cosine) as the lenghts of the legs in the right triangle with hypotenuse 1 and angle α:



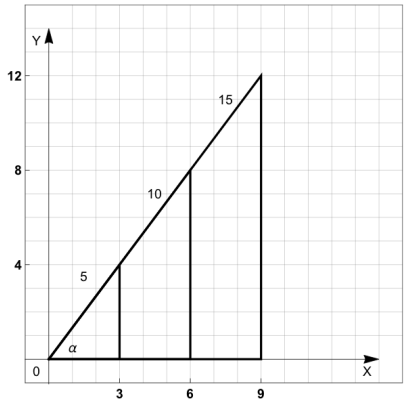
In general, for a right-angle triangle with angle α, we can find $\sin\alpha$ and $\cos\alpha$ by following formulas:

$$\sin\alpha = \frac{\text{opposite side}}{\text{hypotenuse}}$$
$$\cos\alpha = \frac{\text{adjacent side}}{\text{hypotenuse}}$$

Interestingly, the definitions on $\sin$ and $\cos$ do not really depend on size of the triangle, but only the angle itself. Since any two right triangles with the same angles are similar, it shows that if we have a right triangle with angle α and hypotenuse r, then the sides will be $r\sin\alpha$ and $r\cos\alpha$:



For example:



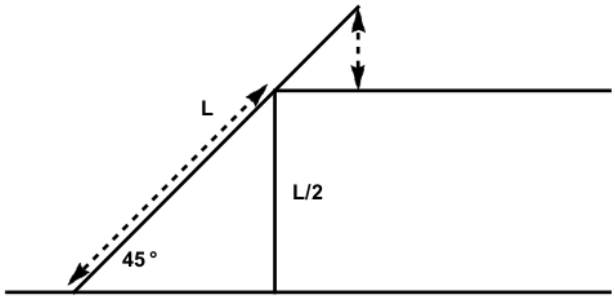
$$\sin\alpha = \frac{\text{opposite side}}{\text{hypotenuse}} = \frac{4}{5} = \frac{8}{10} = \frac{12}{15}$$
$$\cos\alpha = \frac{\text{adjacent side}}{\text{hypotenuse}} = \frac{3}{5} = \frac{6}{10} = \frac{9}{15}$$

There are some special angles, for which $\sin$ and $\cos$ can be computed explicitly:

Trigonometric Functions						
Function	Notation	Definition	$0°$	$30°$	$45°$	$60°$
sine	$\sin(\alpha)$	$\frac{\text{opposite side}}{\text{hypotenuse}}$	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$
cosine	$\cos(\alpha)$	$\frac{\text{adjacent side}}{\text{hypotenuse}}$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$

HOMEWORK

1. Which one is greater?
   (a) 0 or $\sin 0$
   (b) 1 or $\sin 30$
   (c) $\sin 45$ or $\cos 45$
   (d) $\cos 60$ or $\sin 30$
2. A tree casts a 60 m long shadow when the angle of elevation of the sun is 30°. How tall is the tree? [Angle of elevation is the angle that line from tip of shadow on ground to top of tree makes with the horizontal.]
3. A ladder of length L is resting on a ledge whose height is half of the ladder's length. The ladder makes a 45° angle with the ground.
   (a) How long is the portion of the ladder between the ground and the point of contact of ledge and ladder? [indicated by a long dashed arrow]
   (b) At what height is the top of ladder above the ledge? [indicated by short dashed arrow]



4. A cruise ship travels north for 3 miles and then north-west for another 3 miles. How far will it end up from its original position? [North-end is the direction that bisects the angle between north and east.]
5. A ship travels for 3 miles north, then turns and goes for 2 miles northeast, then for another 5 miles north-northeast. Where will it be at the end? how far east and north of the original position? [Northeast means that its direction bisects the angle between north and east directions, thus forming an angle of 45° with due north. North-northeast means that this direction bisects the angle between north and north-east, thus forming 22.5° angle with due north.]
6. Consider a regular pentagon inscribed in a circle of radius 1. What is the side length of such a pentagon? [Hint: drop a perpendicular from the center to one of the sides and complete it to form a right triangle.]
7. Consider a parallelogram $ABCD$ with $AB = 1$, $AD = 3$, $\angle A = 40^\circ$. Find the lengths of diagonals in this parallelogram.
8. Prove that the area of a triangle $\triangle ABC$ can be computed using the formula $A = \frac{1}{2} AB \cdot AC \cdot \sin \angle A$. [Hint: what is the altitude from vertex B?]
9. What is the area of a regular pentagon inscribed in a circle of radius 10? [Make sure to use a trigonometric function.]