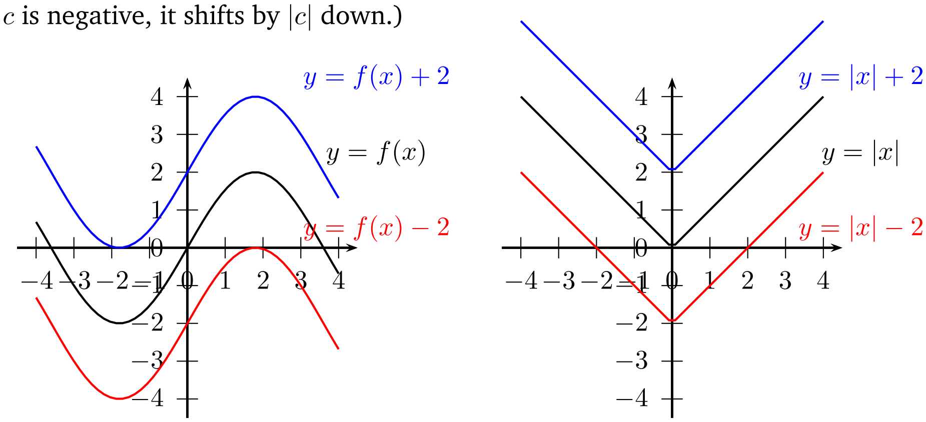
# MATH 7: HANDOUT 18
## COORDINATE GEOMETRY 2: TRANSFORMATION AND MORE BASIC GRAPHS.

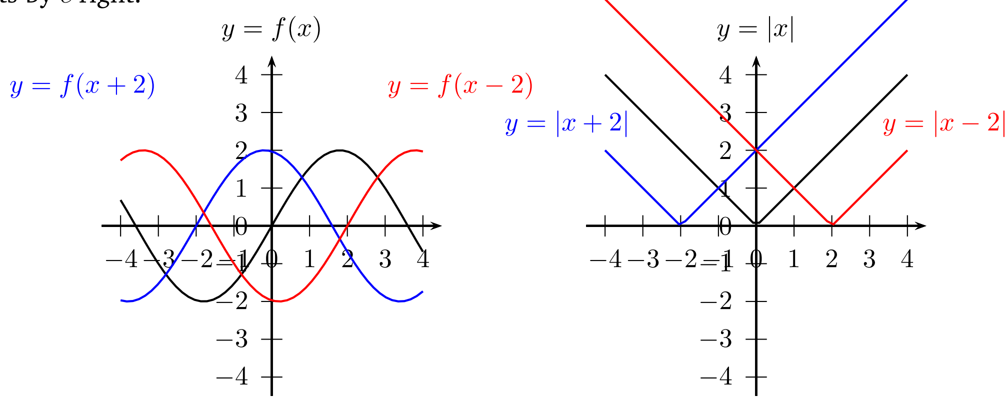
### Transformations

Having learned a number of basic graphs, we can produce new graphs, by doing certain transformations of the equations. Here are two of them.

**Vertical translations:** Adding constant $c$ to the right-hand side of equation shifts the graph by $c$ units up (if $c$ is positive; if $c$ is negative, it shifts by $|c|$ down.)

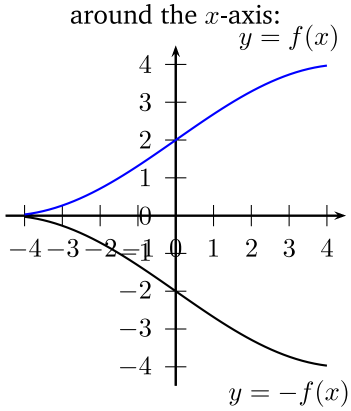



**Horizontal translations:** Adding constant $c$ to $x$ shifts the graph by $c$ units left if $c$ is positive; if $c$ is negative, it shifts by $c$ right.

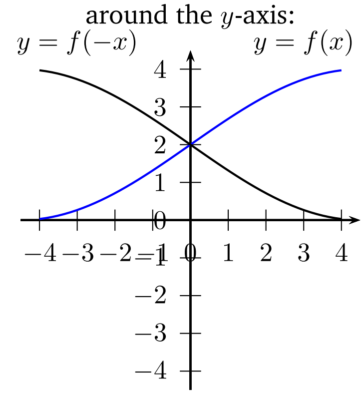



**Reflections**

Multiplying the function by $-1$ reflects the graph around the $x$-axis:



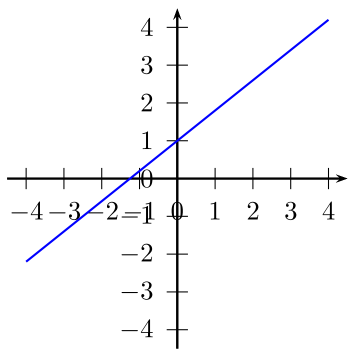

Replacing in the equation $x$ by $-x$ reflects the graph around the $y$-axis:



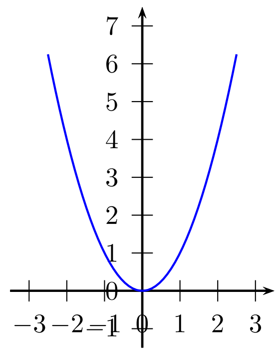
Combining the knowledge of transformations with the knowledge of graphs of basic functions, we can already build a large number of graphs.

**Linear function:** $y = mx + b$

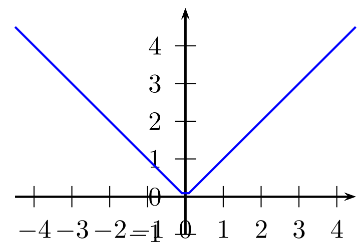
The graph of this function is a straight line. The coefficient $m$ is called the *slope*.



**Parabola:** $y = x^2$

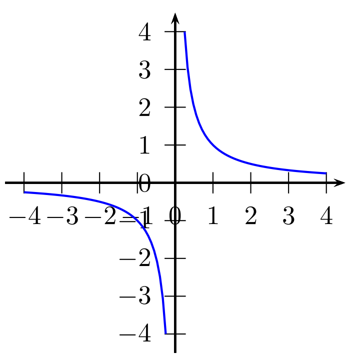


**Absolute value:** $y = |x|$

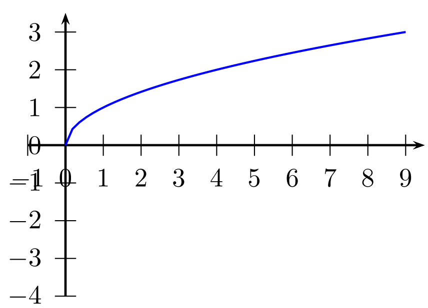


**Inverse function:** $y = \frac{1}{x}$

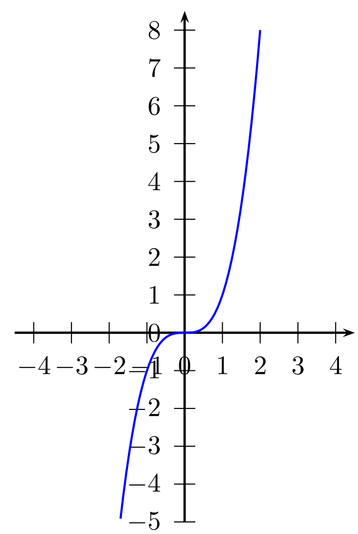
The graph of this function is called a **hyperbola**.



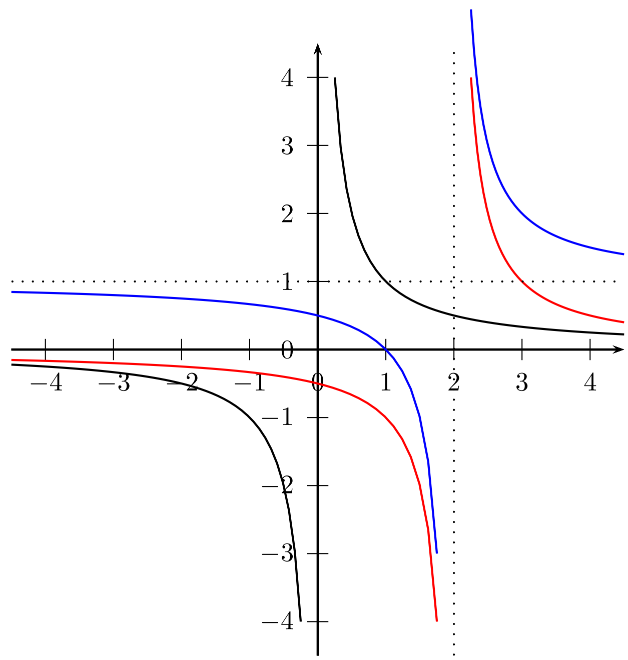
**Square root:** $y = \sqrt{x}$



**Cubic function:** $y = x^3$



Here is an example: plot the graph of the function $y = \frac{1}{x-2} + 1$. We start with the graph of $y = \frac{1}{x}$ (black on the picture below), and then do two translations: first by 2 to the right, to draw $y = \frac{1}{x-2}$ (red), and then by 1 up, to finally get $y = \frac{1}{x-2} + 1$ (blue).




HOMEWORK

**1.** Let $A = (3, 5)$, $B = (6, 1)$ be two of the vertices of a square $ABCD$ (the vertices are labeled $A, B, C, D$ going counterclockwise). Find the coordinates of points $C, D$ and of the center of the square. Find the area of this square.

**2.** Let $C$ be the circle with center at $(0, 1)$ and radius $2$, and $l$ - the line with slope $1$ going through the origin. Find the intersection points of the circle $C$ and line $l$, and compute the distance between them.

***3.** Prove the following formula for the distance from a point to the line: the distance from point $P = (u, v)$ to the line given by equation $ax + by = 0$ is

$$d = \frac{|au + bv|}{\sqrt{a^2 + b^2}}$$

**4.** Prove that for any point $P$ on the parabola $y = \frac{x^2}{4} + 1$, the distance from $P$ to the $x$-axis is equal to the distance from $P$ to the point $(0, 2)$.

**5.** Prove that the set of all points $P$ satisfying the following equation

$$\text{distance from } P \text{ to the origin} = 2 \cdot (\text{distance from } P \text{ to } (0,3))$$

is a circle. Find its radius and center.

**6.** (a) Sketch the graphs of functions $y = |x + 1|$ and $y = -x + 0.25$.
(b) How many solutions do you think this equation has?

$$|x + 1| = -x + 0.25$$

**Note:** you are not asked to find the solutions — just answer how many are there.

**7.** (a) Draw the graph of the equation $x^2 + y^2 - 1 = 0$.
(b) Draw the graph of the equation $x^2 + (y - 1)^2 - 1 = 0$.
(c) Draw the graph of the equation $xy = 0$.
(d) Draw the graph of the equation $x^2 + y^2 = 0$.
(e) Draw the graph of the equation $(x^2 + y^2 - 1)(x^2 + (y - 1)^2 - 1) = 0$.
(f) Draw the graph of the equation $(x^2 + y^2 - 1)^2 + (x^2 + (y - 1)^2 - 1)^2 = 0$.

**8.** Sketch graphs of the following functions:

(a) $y = (x-1)^2 + 1$

(b) $y = \dfrac{1}{x+2} + 1$

(c) $y = \dfrac{1}{2-x}$

(d) $y = \dfrac{x+2}{x+1}$

(e) $y = \left|\dfrac{1}{x-1} + 1\right|$