MATH 7: HANDOUT 2
ALGEBRAIC EXPRESSIONS AND IDENTITIES

MAIN ALGEBRAIC IDENTITIES

$$(a+b)^2 = a^2 + 2ab + b^2$$
$$(a-b)^2 = a^2 - 2ab + b^2$$
$$a^2 - b^2 = (a-b)(a+b)$$

We also talked about the formulas for the third power (cube) of the sum and difference:

$$(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$$
$$(a-b)^3 = a^2 - 3a^2b + 3ab^2 - b^3$$

There are also formulas for a difference of two cubes and for a sum of two cubes. Notice that we did not have a formula for the sum of two squares!!!

$$a^3 - b^3 = (a-b)(a^2 + ab + b^2)$$
$$a^3 + b^3 = (a+b)(a^2 - ab + b^2)$$

HOMEWORK

1. Simplify:

(a) $\sqrt{\dfrac{56}{13}} \cdot \sqrt{\dfrac{26}{7}} =$

(b) $\sqrt{48} =$

(c) $\dfrac{\sqrt{48}}{\sqrt{15}} =$

2. Express the following expressions in the form $2^r 3^s a^m b^n$:

(a) $8a^3b^2(27a^3)(2^5 ab) =$

(b) $3^2(2ab)^3(16a^2b^5)(24b^2a) =$

(c) $16a^2b^3(6ab^4)(ab^2)^3 =$

3. Expand as sums of powers of x:

(a) $(2x+5)^2 =$

(b) $(2-4x)^2 =$

(c) $(1-2x)^2 =$

(d) $(1-x)^2(2-x) =$

(e) $(2x+1)^2(2-3x) =$

4. Factor (i.e., write as a product) the following expressions:

(a) $4x^2 + 8xy + 4y^2$
(b) $9x^2 - 25$
(c) $(x-2)^2 - (y+3)^2$
(d) $(x-2)^2 - 10(x-2) + 25$
(e) $256 - a^8b^8$

(f) $3x^3 - x^2y + 6x^2y - 2xy^2 + 3xy^2 - y^3$
(g) $a^2 - b^2 - 10b - 25$
(h) $x^4 + 4$
    [Hint: add and then subtract $4x^2$.]

5. Solve the following equations.

(a) $5(x+1) = 3x + 2$

(b) $(x^2 - 1)(x + 2) = 0$

(c) $\frac{x+2}{x+3} = 2$

(d) $(x-3)(x+4) = 0$

(e) $x^2 + 4x = 0$

(f) $x^3 + 4x = 0$

6. Find expansions of $(a + b)^4$, $(a - b)^4$ using the previous results.

7. A $4 \times 4 \times 4$ cubical box has 64 small cubes inside. How many of these touch a side or the bottom of the box?

8. Amanda has an average of 92 on her seven tests. What should she get on her $8th$ test to have an average of 93?