Homework 26: Trigonometry, law of sines.

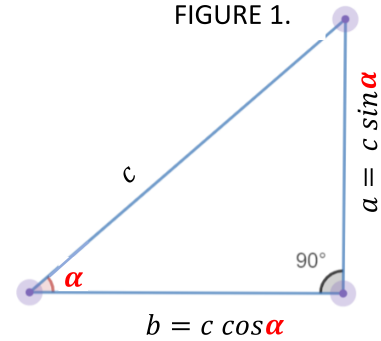
HW26 is Due May 2; submit to Google classroom 15 minutes before the class time.

1. Definition for sin and cos of an angle

In general, for a right-angle triangle, the ***sin*** α and ***cos*** α of the angle are defined as:

$$\sin(\alpha) = \frac{\text{opposite side}}{\text{hypothenuse}} = \frac{a}{c}, \quad \cos(\alpha) = \frac{\text{adjacent side}}{\text{hypothenuse}} = \frac{b}{c}$$

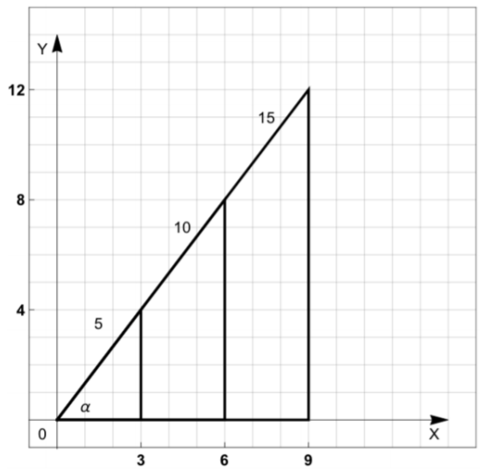
FIGURE 1.



2. Definition of tangent of an angle

Now we can also define the 3rd trigonometric ratio:

$$\tan(\alpha) = \frac{\sin(\alpha)}{\cos(\alpha)} = \frac{\text{opposite side/hypothenuse}}{\text{adjacent side/hypothenuse}} = \frac{a}{b}$$



Example: Consider the angle a in the following triangles:

$$\sin(\alpha) = \frac{\text{opposite side}}{\text{hypothenuse}} = \frac{4}{5} = \frac{8}{10} = \frac{12}{15}$$

$$\cos(\alpha) = \frac{\text{adjacent side}}{\text{hypothenuse}} = \frac{3}{5} = \frac{6}{10} = \frac{9}{15}$$

$$\tan(\alpha) = \frac{\text{opposite side}}{\text{adjacent side}} = \frac{4}{3} = \frac{8}{6} = \frac{12}{9}$$

3. Table with values for trigonometric functions

Function	Notation	Definition	0^0	30^0	45^0	60^0	90^0
sine	$\sin(\alpha)$	$\frac{\text{opposite side}}{\text{hypothenuse}}$	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1
cosine	$\cos(\alpha)$	$\frac{\text{adjacent side}}{\text{hypothenuse}}$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0
tan	$\tan(\alpha)$	$\frac{\text{opposite side}}{\text{adjacent side}}$	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	∞

4. Trigonometric identities and the law of sines

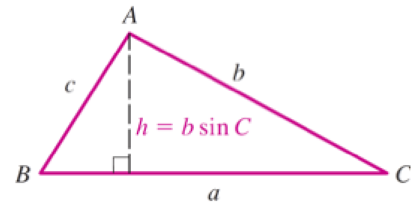
- The most prominent trigonometric identity: $\mathbf{\sin^2 \alpha + \cos^2 \alpha = 1}$

Proof: Pythagoras theorem for Fig1 gives $a^2 + b^2 = c^2$; $(c\sin(\alpha))^2 + (c\cos(\alpha))^2 = c^2$; then divide both sides by c^2 to obtain the identity.

- The law of sines: Given a triangle ΔABC with sides *a*, *b*, and c, the following is always true:

$$\frac{\boldsymbol{a}}{\boldsymbol{sin(A)}} = \frac{\boldsymbol{b}}{\mathbf{sin}(\boldsymbol{B})} = \frac{\boldsymbol{c}}{\mathbf{sin}(\boldsymbol{C})}$$

Proof: To see why the Law of Sines is true, refer to the figure. The height of the triangle $h = b\sin(C)$, and therefore the area is: $S = \frac{1}{2}a \times b\sin(C)$. Similarly, $h = c\sin(A)$ and $S = \frac{1}{2}a \times c\sin(B)$. Constructing a height towards side b, $S = \frac{1}{2}b \times c\sin(A)$. Thus, $bc\sin(A) = ac\sin(B) = ab\sin(C)$. Divide by *abc*, to get the law.



Homework problems

Instructions: Please always write solutions on a ***separate sheet of paper***. Solutions should include explanations. I want to see more than just an answer: I also want to see how you arrived at this answer, and some justification why this is indeed the answer. So **please include sufficient explanations**, which should be clearly written so that I can read them and follow your arguments.

All angles are measured in degrees.

1. If a right triangle ΔABC has sides $AB = 3 \times \sqrt{3}$ and $BC = 9$, and side AC is the hypotenuse, find all 3 angles of the triangle.
2. The area of a right triangle is 36 square meters. The legs of the triangle have the ratio of 2 : 9. Find the hypotenuse of the triangle.
3. In a triangle ΔABC, we have $<$ A = 40^0; $<$ B = 60^0, and AB = 2 cm. What is the remaining angle and side lengths? (Hint: Use Law of sines)
4. In an isosceles triangle, the angle between the equal sides is equal to 30^0, and the height is 8. Find the sides of the triangle.
5. A right triangle ΔABC is positioned such that A is at the origin, B is in the 1st quadrant (coordinates B*x* > 0 and By > 0) and C is on the positive horizontal axis (C*x* > 0 and Cy = 0). If length of side AB is 1, and AB makes a 35^0 angle with positive x-axis, what are the coordinates of B?
6. Consider a parallelogram ABCD with AB = 10, AD = 4 and $<$ BAD = 50^0. Find the length of diagonal AC. (Hint: make ΔACM, where $< M$ is 90^0 and point M is on the same line as CD)
7. A regular heptagon (7 sides) is inscribed into a circle of radius 1.
   a. What is the perimeter of the heptagon?
   b. What is the area of the heptagon?
8. In the trapezoid below, AD = 5 cm, AB = 2 cm, and $<$ A = $<$ D = 70^0. Find the length BC and the diagonals. [You can use: $\sin(70^0) \approx 0{:}94$; $\cos(70^0) \approx 0{:}34$.]
9. To determine the distance to the enemy gun (point E in the Figure 3 below), the army unit placed two observers (points A, B in the figure below) and asked each of them to measure the angles using a special instrument. The results of the measurements are shown below. If it is known that the distance between the observers is 400 meters, can you determine how far away from observer A is the enemy gun?

FIGURE 2.

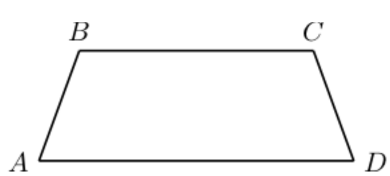


FIGURE 3.