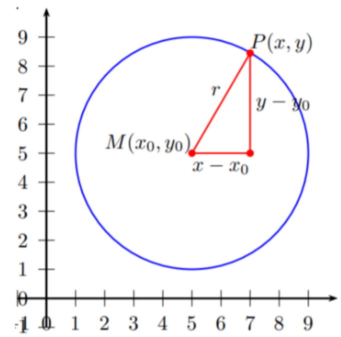
Homework 24: Coordinate geometry, adding graphs.

HW24 is Due April 18; submit to Google classroom 15 minutes before the class time.

1. Coordinate geometry: Introduction



The **midpoint** M of a segment AB with endpoints A(x_1, y_1) and B(x_2, y_2) has coordinates:

$$M\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$$

The distance between two points $A(x_1, y_1)$ and $B(x_2, y_2)$ is given by the following formula:

$$\boldsymbol{d = \sqrt{(x_2 - x_2)^2 + (y_2 - y_1)^2}}$$

Line: $y = \boldsymbol{m}x + \boldsymbol{b}$

with a **slope** $m = \frac{\Delta y}{\Delta x} = \frac{y_2-y_1}{x_2-x_1}$ and intercept ***b.***

Parabola: $y = ax^2 + bx + c$ (standard form) or $y = a(x-h)^2 + \mathrm{k}$ (vertex form)

Circles: The equation of the circle with the center $M(x_0, y_0)$ and radius $\boldsymbol{r}$ is: $(x - x_0)^2 + (y - y_0)^2 = r^2$.

2. Graphs of functions

In general, the relation between x and y could be more complicated and could be given by some formula of the form $y = f(x)$, where f is some function of x (i.e., some formula which contains x). Then the set of all points whose coordinates satisfy this relation is called the **graph** of f.

- Vertical shift: $y = f(x) + \boldsymbol{k}$, shift up by k $\quad y = f(x) - \boldsymbol{k}$, shift down by k $\quad$ $\mathrm{k} > 0$
- Horizontal shift: $y = f(x-h)$, shift right by h $\quad y = f(x+h)$, shift to the left by h $\quad$ $h > 0$
- Reflection, x-axis: $y = -f(x)$, multiply the function by -1.
- Reflection, y-axis: $y = f(-x)$, multiply the argument by -1.

3. Quadratic function (revisited +)

Quadratic equation in a standard form: $ax^2 + bx + c = 0$

- a, b, c – coefficients, determinant D: $\boldsymbol{D = b^2 - 4ac}$, solutions(roots): $\boldsymbol{x_{1,2} = \frac{-b \pm \sqrt{D}}{2a}}$
- **D** determines the number of roots! ($D < 0$ no solutions, $D = 0$ one solution, $D > 0$ two solutions)

Quadratic function in a factored form: $y = a(x - x_1)(x - x_2)$, where

- roots: the numbers x_1 and x_2 – solutions of the quadratic equation ($y = 0$)
- **Vieta's formulas:** The roots are related to the coefficients: $\boldsymbol{x_1 x_2 = \frac{c}{a}}$ and $\boldsymbol{x_1 + x_2 = -\frac{b}{a}}$

Quadratic function in a vertex form: $y = \boldsymbol{a}(\boldsymbol{x} - \boldsymbol{h})^2 + \boldsymbol{k}$

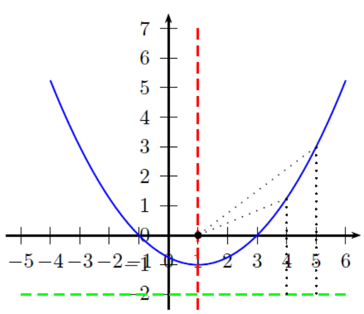
- **Method 1: completing the square**. Use the formulas for fast multiplication.
- **Method 2: find the vertex**. Determine the coefficients a, b, c. Find the vertex x-and y- coordinates

  $\boldsymbol{x_v = h = -\frac{b}{2a}}$. $\qquad \boldsymbol{y_v = k} = y(\boldsymbol{x_v}) = a\boldsymbol{x_v}^2 + b\boldsymbol{x_v} + c$

Modified vertex form: rewrite the equation into separate $y-$ and $x-$ part $4\boldsymbol{p}(y - \boldsymbol{k}) = (x - \boldsymbol{h})^2$

Distance from any point on the parabola to focus and directrix: $\boldsymbol{p} = \frac{1}{4a}$

Vertex $V(h,k)$ Focus $F(h, k+\boldsymbol{p})$ directrix $y = k - \boldsymbol{p}$



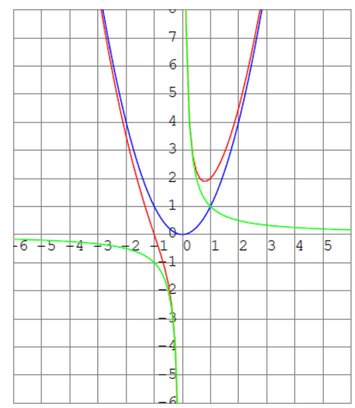
NEW: Parabola is the set of all points in a plane that are equally distant away from a given point and a given line (see black dotted lines). This given point is called **the focus** (black dot) of the parabola and the line is called **the directrix** (green line).

- If the parabola is of the form $(x-h)^2 = 4p(y-k)$, the vertex is $(h.k)$, the focus is $(h, k+p)$ and directrix is $y = k - p$.

4. Adding Graphs

Now that we know how to draw a lot of basic graphs and how to use transformations, we can draw more complicated graphs — that is, graphs that we get by adding two functions. For example, if we want to draw a graph of a function $y = x^2 + \frac{1}{x}$

We can carefully examine two separate graphs of $y = x^2$ (**blue**) and $y = \frac{1}{x}$ (**green**), and then see what happens if one adds these two graphs (**red**) by adding their y-values for every x.



Homework problems

Instructions: Please always write solutions on a ***separate sheet of paper***. Solutions should include explanations. I want to see more than just an answer: I also want to see how you arrived at this answer, and some justification why this is indeed the answer. So **please include sufficient explanations**, which should be clearly written so that I can read them and follow your arguments.

ALL GRAPHS/POINTS/FIGURES SHOULD BE DRAWN BY YOU - NOT PRINTED!   USE QUADRILE PAPER!

1. Graph $x^2 = 4y$. What is the focus, directrix and vertex of the parabola?

2. Sketch the following functions by first drawing the graph of each addend function and then adding the y-values for a few x-values. (Review your class notes)
   a. $y = |x| + |x+1|$
   b. $y = |x-1| + |x+1|$
   c. $y = |x-1| - |x+1|$
   d. $|y| = x$ (Hint: what is the domain/range of this function?)

3. Sketch the following functions by first drawing the graph of each addend function and then adding the y-values for a few x-values.
   a. $y = x + \frac{1}{|x|}$
   b. $y = \sqrt{x} + \frac{1}{x}$
   c. $y = x - \frac{1}{x^2}$

4. Find all intersection points of the parabola $y = x^2$ and the circle with radius $\sqrt{6}$ and center at (0,4).