Homework 18: Inequalities, quadratic functions in vertex form

HW18 is Due February 28; submit to Google classroom 15 minutes before the class time.

1. Converting from standard to vertex form

We know how to draw the graph of $y = x^2$. It's a parabola.

- We know that the graph of $y = x^2 + k$ can be obtained from the graph of $y = x^2$ by shifting up by k units (or down, if k < 0)
- We know that the graph of $y = (x + \mathrm{h})^2$ can be obtained from the graph of $y = x^2$ by shifting left by h units (or right, if h < 0).
- Based on the two facts above, we can draw a graph of any function of the type $y = (x + h)^2 + k$. The values $(-h, k)$ are the coordinates of the vertex of the parabola.

In general. we can transform any quadratic function $\boldsymbol{y = ax^2 + bx + c}$ to $\boldsymbol{y = a(x + h)^2 + k}$.
This is called transforming from standard into vertex form. The coefficient a has the same value in both forms. Note that, if $b = 0$, then your equation in standard form is already in a vertex form with vertex coordinates $(0, k = c)$.

We will use two ways to convert a quadratic function from standard into vertex form:

- **Method 1: completing the square**. We have learned how to do this using the formulas for fast multiplication.
  <u>*Example:*</u> $y = 2x^2 + 4x - 2 = 2[\boldsymbol{x^2 + 2x} - 1] = 2\left[x^2 + 2.x.1 + 1^2 - 1^2 - 1\right] = 2[(x+1)^2 - 1 - 1] = 2[(x+1)^2 - 2] = 2(x+1)^2 - 4$.

- **Method 2: find the vertex**. Determine the coefficients a, b, c. Find the vertex *x-coordinate* $\boldsymbol{x_v = -h = -\frac{b}{2a}}$. Then, substitute $\boldsymbol{x_v}$ in the equation you are converting and solve for y, $y = a\boldsymbol{x_v}^2 + b\boldsymbol{x_v} + c$. The found value is the vertex *y-coordinate,* $\boldsymbol{y_v = k}$. Write the equation in a vertex form $a(x + h)^2 + k = \mathrm{y}$.
  <u>*Example:*</u> $y = 2x^2 + 4x - 2, \quad a = 2,\ b = 4,\ c = -2$

  Vertex *x-coordinate:* $x_v = -h = -\frac{b}{2a} = -\frac{4}{2.2} = -1$ **But** $h = 1$!
  Vertex *y-coordinate:* $y_v = 2{x_v}^2 + 4x_v - 2 = 2(-1)^2 + 4(-1) - 2 = 2 - 4 - 2 = -4, \quad k = -4$

  New function: to $y = a(x + \boldsymbol{h})^2 + \boldsymbol{k} = 2(x+1)^2 - 4$

Homework problems

Instructions: Please always write solutions on a ***separate sheet of paper***. Solutions should include explanations. I want to see more than just an answer: I also want to see how you arrived at this answer, and some justification why this is indeed the answer. So **please include sufficient explanations**, which should be clearly written so that I can read them and follow your arguments.

1. Solve the following quadratic inequalities:
   a. $2x^2 - 3x + 1 > 0$ (Convert to factored form, see your class notes!)
   b. $4(x - \sqrt{3})(x - 4) \le 0$
   c. $\frac{x}{2x-3} \ge \frac{1-x}{2x-3}$.

2. For each part a), b), c), d), e), and f), graph all functions in desmos.com (you could graph more than one function at a time). Each part studies a different parameter (For example, in a) only the parameter a≠ 0, the other two are set to zero, b = 0, c = 0). At the end, explain the meaning of each parameter in the quadratic function when the function is written in a standard form: $y = ax^2 + bx + c$. You do not need to attach the graphs, but you may ***sketch*** the graphs for each part on the same coordinate system!

   a. Study parameter $a \neq 0$: $y = x^2$, $y = 2x^2$, $y = 5x^2$
   b. Study parameter $a \neq 0$: $y = x^2$, $y = -x^2$, $y = -2x^2$
   c. Study parameter $a \neq 0$: $y = x^2$, $y = \frac{2}{3}x^2$, $y = \frac{1}{3}x^2$
   d. Study parameter $c \neq 0$ (also $a \neq 0$): $y = x^2 + 5$, $y = x^2 - 3$
   e. Study parameter $b \neq 0$ (also $a \neq 0$): $y = x^2 - 4x$, $y = x^2 + 4x$
   f. Study parameter $c \neq 0$ (also $a \neq 0$, $b \neq 0$): $y = x^2 - 4x + 5$, $y = x^2 + 4x + 5$

   At the end, **explain the meaning of each parameter** a, b, c in the quadratic function when the function is written in a standard form: $y = ax^2 + bx + c$.
   Explain means write sentences, equalities, inequalities or anything to show what you have discovered.

3. Use completing the square method (Method 1 shown on page 1 here or see your class notes).
   a) $y = x^2 - 5x + 5$
   b) $y = x^2 - 4x + 2$
   c) $y = x^2 - x - 1$
   d) $y = -x^2 + 3x - 0.5$
   e) $y = x^2 + 4x - 45$