

MATH 6: ASSIGNMENT 14

ARITHMETIC SEQUENCE

A sequence of numbers is an arithmetic sequence if the difference between consecutive terms is the same number, the **common difference**, let's call it d .

For example, let's consider the sequence: 1, 5, 9, 13, 17, ...

The first term in the sequence is $a_1 = 1$, the second is $a_2 = 5$, and so on. The difference is $d = 4$.

What is the n^{th} term? For example what is a_{100} ?

$$a_1 = 1$$

$$a_2 = a_1 + d = 1 + 4 = 5$$

$$a_3 = a_2 + d = (a_1 + d) + d = a_1 + 2d = (1 + 4) + 4 = 1 + 2 \times 4 = 9$$

$$a_4 = a_3 + d = (a_2 + d) + d = ((a_1 + d) + d) + d = a_1 + 3d = 1 + 3 \times 4 = 13$$

....

$$a_n = a_1 + (n - 1)d$$

$$\text{So } a_{100} = a_1 + 99d = 1 + 99 \times 4 = 397$$

PROPERTY OF AN ARITHMETIC SEQUENCE

A property of an arithmetic sequence is that any term is the arithmetic mean of its neighbors.

$$a_n = \frac{a_{n-1} + a_{n+1}}{2}$$

To prove this we can write:

$$\begin{aligned} a_n &= a_{n-1} + d \\ a_n &= a_{n+1} - d \end{aligned}$$

Add them up and we have $2a_n = a_{n-1} + a_{n+1}$, divide by 2 and we get $a_n = \frac{a_{n-1} + a_{n+1}}{2}$

To find the common difference between 2 terms a_s and a_t

$$d = \frac{a_s - a_t}{s - t}.$$

SUM OF AN ARITHMETIC SEQUENCE

$$S = a_1 + a_2 + a_3 + \dots + a_n = n \times \frac{a_1 + a_n}{2}$$

To prove this, we write the sum in 2 ways, in increasing and decreasing order:

$$S = a_1 + a_2 + a_3 + \dots + a_n$$

$$S = a_n + a_{n-1} + a_{n-2} + \dots + a_1$$

Adding up, we notice that $a_1 + a_n = a_2 + a_{n-1} = a_3 + a_{n-2} = \dots$

$$2S = (a_1 + a_n) \times n \text{ and dividing by 2 gives us } S = n \times \frac{a_1 + a_n}{2}.$$

HOMEWORK PROBLEMS

1. Write the first 5 terms of an arithmetic sequence if $a_1 = 7$ and $d = 2$.
2. What are the first 2 terms for the sequence $a_1, a_2, -9, -2, 5, \dots$
3. $a_{10} = 131$ and $d = 12$. What is a_1 ?
4. $a_5 = 27$ and $a_{27} = 60$. Find the first term and the common difference.
5. Find the common difference in an arithmetic sequence if the 9^{th} term is 18 and the 11^{th} term is 44.
6. Find the sum of the first 100 terms if $a_1 = 10$ and $a_{100} = 150$.
7. Find the sum of all odd numbers from 1 to 2011.
- *8. Can you continue the following sequence: $-2, 1, 6, 13, 22, \dots$ [Hint: look at the differences of successive terms!]
9. Find the shortest distance from the origin $(0,0)$ to the line given by the equation $y = -2x + 8$.
10. Compute the area of the rectangle $ABCD$ if A is at $(0,0)$, B at $(2,3)$, and D at $(-6,8)$. [It can be done in more than one way.]