MATH 5 — PREREQUISITES FOR MATH 6

1. Program

- Algebraic expressions. Commutativity, associativity, distributivity.
- Equations. Solving word problems with equations.
- Powers of 2.
- Binary numbers.
- Powers. Negative powers. Scientific notation.
- $a^2 - b^2 = (a - b)(a + b)$
- Square roots.
- Pythagorean theorem.
- Basic probability theory: addition rule, complement rule, product rule.
- Geometry: parallel lines and angles (alternate interior, alternate exterior, corresponding).
- Parallelogram, various definitions, properties.
- Congruence tests for triangles (SAS, ASA, SSS).
- Isosceles triangle. Median, bisector, height.
- Trapezoid. Its midline. Area.

2. Problems

1. Rewrite each of the expressions below in the simplest possible form, by collecting the like terms if possible.

$$2x + 7 + 5x + 2 + 3x \qquad 3x + 9 + 5xy + 2xy + 3 \qquad 3(2x - 1) + x$$
$$2a(a - 2) - a(a - 1) \qquad (2x - 1)(x + 1)$$

2. An apple cost 9 cents, and an orange 15 cents. Elena bought some apples and oranges, 20 fruit in all, and paid \$2.64. How many apples and how many oranges did she buy?

3. A boy had a bag of apples. He gave 1/2 of them to his parents, 1/5 to his brother, 1/4 to his sister and the last apple he ate himself. How many apples did he originally have?

4. Simplify the following expressions

(a) $x + 4(1 - x)$ (b) $2 + 5x - 4(3 - x)$ (c) $5(x - 1) - 3(2x + 1)$

5. If you take half my age and add 7, you get my age 13 years ago. How old am I?

6. Two secretaries, Barbara and Mary, need to type a 100 page document. Barbara can type it in 4 hours; Mary types slower, so it would take her 5 hours to do this. How fast can they type it together if they divide the work between two of them in the most efficient way?

7. Find the sum $1 + 2 + 4 + \cdots + 2^n$ (the answer, of course, will depend on n). [Hint: first try computing it for several small values of n: find $1 + 2$, then $1 + 2 + 4$, then $1 + 2 + 4 + 8$. See if you can notice a pattern. After this, try formulating a general rule.]

8. Convert the decimal numbers to binary:
9, 12, 24, 38, 45

9. Convert the following binary numbers to decimal:
101, 1001, 10110, 11011, 10101

10. Compute $110101_b + 111011_b$ without converting numbers to decimal form.

11. Simplify the following expression:

$$\frac{(x^2y^2) \cdot x^3}{x^2y^5}$$

12. Let $a = 2 \cdot 10^8$, $b = 10^5$. Compute $a^2 \cdot b$, $\frac{a}{b}$, $a^2 \div b^3$.

13. If $a = 2^{-13}3^9$, $b = 2^{11}3^{-7}$, what is the value of ab? of a/b?

14. Write the following numbers using scientific notation.
(a) the distance from Earth to Pluto is $\approx$ 7,527,000,000 km;
(b) the distance from Earth to the star Sirius is $\approx$ 81,900,000,000,000 km;

15. Factor the following number into primes: $99^2 - 9^2$. [Hint: you do not have to compute this number.]

16. Find the following square roots. If you can not find the number exactly, at least say between which two whole numbers the answer is, e.g., between 5 and 6.
(a) $\sqrt{81}$
(b) $\sqrt{10,000}$
(c) $\sqrt{10^8}$

17. If, in a right triangle, one leg has length 1 and the hypotenuse has length 2, what is the other leg?

18. Simplify: $(\sqrt{17})^2$, $(\sqrt{13})^4$, $(\sqrt{11})^3$, $\sqrt{2^4\,3^6}$, $\sqrt{2^4\,3^5}$.

19. A license plate consists of 3 letters, followed by three digits. How many possible license plates are there?

20. We roll two identical dice. What is the probability of getting 1 and 3?

21. We roll two identical dice. What is the probability of getting sum of two numbers equal to 4?

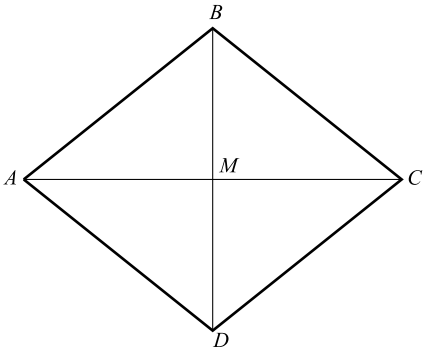
22. If we toss a coin 5 times, what is the probability that **at least one** will be heads?

23. Solve equations:

(a) $\frac{3}{8}x = \frac{1}{3}$ (b) $|2x - 5| = 1$ (c) $\frac{(x-2)}{x-1} = 3$

24. Show that in a parallelogram, diagonally opposite angles are equal $\angle A = \angle C$, $\angle B = \angle D$

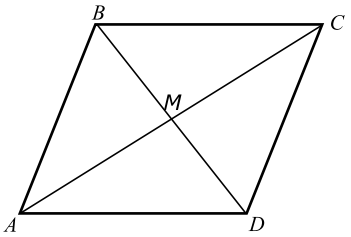
25. Let $ABCD$ be a quadrilateral such that $AB = BC = CD = AD$ (such a quadrilateral is called rhombus). Let M be the intersection point of AC and BD.



(a) Show that $\triangle ABC \cong \triangle ADC$
(b) Show that $\triangle AMB \cong \triangle AMD$
(c) Show that the diagonals are perpendicular and that the point M is the midpoint of each of the diagonals.

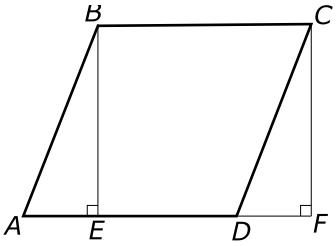
[Hint: after doing each part, mark on the figure all the information you have found — which angles are equal, which line segments are equal, etc: you may need this information for the following parts.]

26. Let $ABCD$ be a quadrilateral such that sides AB and CD are parallel and equal (but we do not know whether sides AD and BC are parallel).



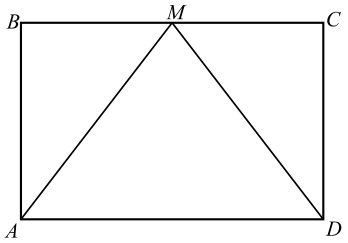
(a) Show that triangles $\triangle AMB$ and $\triangle CMD$ are congruent.
(b) Show that sides AD and BC are indeed parallel and therefore $ABCD$ is a parallelogram.

27. Let $ABCD$ be a parallelogram, and let BE, CF be perpendiculars from B, C to the line AD.



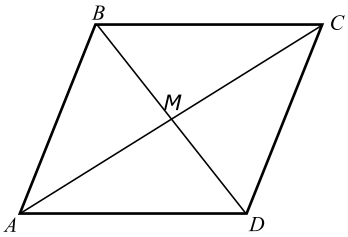
(a) Show that triangles $\triangle ABE$ and $\triangle DCF$ are congruent.
(b) Show that the area of parallelogram is equal to height×base, i.e. $BE \times AD$.

28. In the figure to the right, $ABCD$ is a rectangle, and M is the midpoint of BC. Prove that then triangle AMD is isosceles.



29. (a) Show that if in a quadrilateral $ABCD$ diagonals bisect each other (i.e., intersection point is hte midpoint of each of the diagonals), then $ABCD$ is a parallelogram. [Hint: find some congruent triangles in the figure.]

(b) Show that if in a quadrilateral $ABCD$ diagonals bisect each other and are perpendicular, then it is a rhombus.



30. Find all lenghts, angles, and area in the figure shown to the right.