MATH 6: HANDOUT 20 GEOMETRY

Today we started the study of Euclidean geometry. Some of the material covered is summarized in this handout. We will be using the textbook, *E-Z Geometry*, by Lawrence Leff. You can get a copy at amazon.com.

For your enjoyment, take a look at the book which gave rise to Euclidean geometry and much more, Euclid's *Elements*, dated about 300 BC, and used as the standard textbook for the next 2000 years. Nowadays it is avaialable online at `http://math.clarku.edu/~djoyce/java/elements/toc.html`

1. Basic objects

These objects are the basis of all our constructions: all objects we will be discussing will be defined in terms of these objects. No definition is given for these basic objects.

- Points
- Lines
- Distances: for any two points A, B, there is a non-negative number AB, called distance between A, B.
- Angle measures: for any angle $\angle ABC$, there is a real number $m\angle ABC$, called the measure of this angle (more on this later).

We will also frequently use words "between" when describing relative position of points on a line (as in: A is between B and C) and "inside" (as in: point C is inside angle $\angle AOB$).

Having these basic notions, we can now define more objects. Namely, we can give definitions of

- interval, or line segment (notation: $\overline{AB}$)
- ray (notation: $\overrightarrow{AB}$)
- angle (notation: $\angle AOD$)
- parallel lines: two distinct lines l, m are called parallel (notation: $l \parallel m$) if they do not intersect, i.e. have no common points

2. Supplementary and Complementary Angles

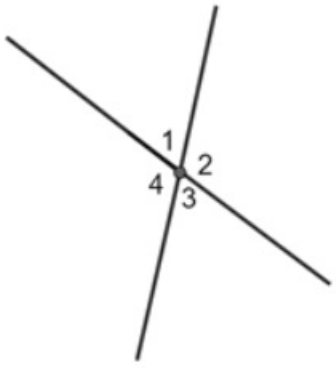
Two angles are **supplementary** if the sum of their measures is 180°. If angle A is supplementary to angle B, then m∠A + m∠B = 180°, and each angle is called the supplement of the other angle.
Two angles are **complementary** if the sum of their measures is 90°. If angle A is complementary to angle B, then m∠A + m∠B = 90°, and each angle is called the complement of the other angle.

3. Two points Determine a Line

For any two points there is a unique line that contains these two points.

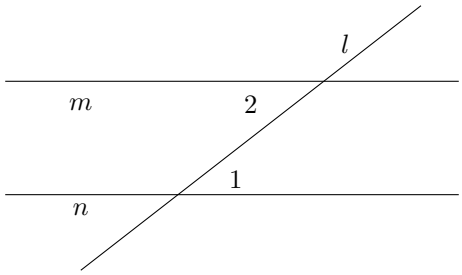
4. Vertical Angles

Two lines that intersect form two pairs of congruent angles.



Angles $\angle 1 = \angle 3$ and $\angle 2 = \angle 4$ by Vertical Angles (or Opposite Angles).
To prove this, notice that m∠ 1 + m∠ 2=180°, also m∠ 1 + m∠ 4 = 180°. So m∠ 2 = m∠ 4.

5. Alternate Interior Angles



Given m $\parallel$ n, then m$\angle$ 1 = m$\angle$ 2.

6. Congruent alternate interior angles implies Parallel Lines

The converse to the alternate interior angles postulate is also true. If the alternate interior angles are congruent, then the two lines are parallel. This postulate can be used to prove that two lines are parallel.

7. Euclid's Parallel Line

Given a line and a point not on the line, at most one line parallel to the given line can be drawn through the point.

8. Homework

1. What is the measure of an angle, if it is larger than twice the measure of its complement by 30°.
2. Show that if two angles are congruent and supplementary, then each is a right angle.
3. Show that if two lines intersect and form congruent adjacent angles, then the lines are perpendicular.
4. Exercises 3,4,5 on page 55 in the book.
5. Exercise 10 on pages 56 in the book.
6. Exercises 4,5 on page 76 in the book.
7. Suppose we draw k lines on the plane so that each of them intersects each other, and all intersection points are distinct. Into how many pieces will they cut the plane? [Hint: how does the number of pieces change when you increase k by 1, i.e. add one more line?]