MATH 6
HANDOUT 18: SOLVING EQUATIONS AND INEQUALITIES

Solving inequalities involving negative numbers

Today we discussed inequalities and their solutions, discussing how one solves inequalities involving negative numbers. The rule here is

$$a < b \iff (-a) > (-b)$$

(note that the sign of inequality is reversed!). For example, $3 < 5$, but $-3 > -5$.

More generally, if we multiply or divide both sides of an inequality by a negative number, we need to change the sign of inequaltiy, replacing $<$ by $>$ and vice versa. For example, to solve $-3x > -6$, we divide both sides by -3 and change $>$ to $<$, giving $x < 2$.

Products

The following rules are frequently used when dealing with equations or inequalities where the left-hand side is a product of two factors:

- A product of two numbers is zero if and only if one of them is zero:

$$ab = 0 \iff (a = 0) \text{ or } (b = 0)$$

- A product of two numbers is positive if and only if both numbers are positive or both numbers are negative:

$$ab > 0 \iff (a > 0 \text{ and } b > 0) \text{ or } (a < 0 \text{ and } b < 0)$$

- A product of two numbers is negative if and only if one of numbers is positive and the other one is negative:

$$ab < 0 \iff (a > 0 \text{ and } b < 0) \text{ or } (a < 0 \text{ and } b > 0)$$

Homework

1. (This problem is for students who have some difficulty with negative numbers and need extra practice).

Compute the following expressions:

$-3-(-2)$	$-3-3$	$((-4)\times 6)-7$
$(-6)\div(-2)+3$	$(-2)\div(-3)$	$(-4)\times(-7)\div(+9)$
$(-4)-(-3)+(-5)$	$(-6)+(-2)\div(-3)$	

2. Solve the following inequalities

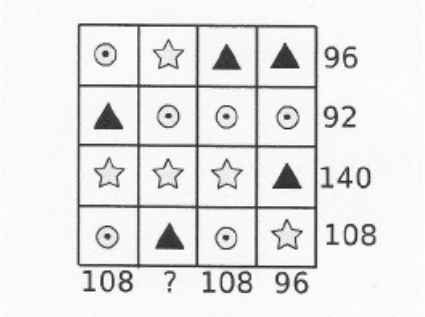
(a) $-x < 2$	(b) $2-3x > 5$	(c) $3x+1 < 5x+7$
(d) $1+5x < 3x$	(e) $x-1 < x-7$	

3. Solve the following equations and inequalities:

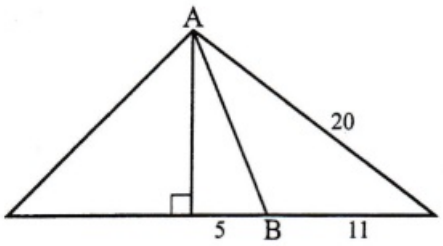
(a) $(x-1)(x-2) = 0$ (b) $x(x+1) < 0$ (c) $\frac{1}{x} > 2$ [Hint: multiply by x]

(d) $x^2-4 = 0$ (e) $\frac{x}{x+1} > 1$ [Hint: consider separately two cases: $x+1 > 0$ and $x+1 < 0$]

4. Graph $y = |x + 1|$.
5. In the figure below, each symbol stands for a number. The sum of numbers in each column or row is written next to the column or row — except for the second column, where the sum is not known. Can you find this missing sum?



6. Old MacDonald raises sheep and chickens on his farm. His livestock has a total of 55 heads and 142 legs among them (not counting the farmer!). How many chickens and how many sheep does he have?
7. Mr. Sim jogs at 9 km per hour over a certain distance and walks back the same distance at 6 km per hour. What is his average speed? [Hint: the answer $(6 + 9)/2 = 7.5$ is wrong!).
8. Show that in a class of 40 students, there must be two students whose birthdays are less than 10 days apart. ‘
9. January 1, 2018 was a Monday. What is the next year in which January 1 will fall on a Monday? (Mathcounts)
10. Find AB (Mathcounts).



11. Three cubes are stacked as shown. If the cubes have edge 1,2 and 3 what is AB? (Mathcounts)