# MATH 6: COORDINATE GEOMETRY

#### COORDINATE GEOMETRY

In the coordinate plane, the general equation of a line is y = mx + b, where m=slope, b=y-intercept.

#### Equation of a Line given the slope and 1 point

If the point has coordinates  $(x_1, y_1)$  and the slope is m, then the equation of the line is  $y - y_1 = m(x - x_1)$ . This equation can be derived using the slope intercept form y = mx + b and the fact that  $(x_1, y_1)$  is on this line, then we can plug in this point and get  $y_1 = mx_1 + b$ , so  $b = y_1 - mx_1$ . Substitute b and we get  $y = mx + y_1 - mx_1$  and  $y - y_1 = mx - mx_1 = m(x - x_1)$ , so  $y - y_1 = m(x - x_1)$ 

# Equation of a Line Given 2 Points

The slope of a line that contains two points with coordinates  $(x_1, y_1)$  and  $(x_2, y_2)$  is  $m = \frac{y_2 - y_1}{x_2 - x_1}$ . The equation of the line containing the two points is:  $y - y_1 = m(x - x_1)$  or  $y - y_2 = m(x - x_2)$ . Choose the point that is easiest to work with.

#### DISTANCE BETWEEN TWO POINTS



The distance between two points  $(x_1, y_1)$  and  $(x_2, y_2)$  can be calculated using Pythagoras theorem in the given right triangle. The hypotenuse is d and we have  $d^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$  and  $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ 

# MIDPOINT OF A LINE SEGMENT

The midpoint of a segment with endpoints  $(x_1, y_1)$  and  $(x_2, y_2)$  is the point with coordinates  $(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2})$ 

# PARALLEL AND PERPENDICULAR LINES

If two lines are parallel, then their slopes are equal. If lines having slopes  $m_1$  and  $m_2$  are perpendicular, then  $m_1m_2 = -1$ 

### Homework

- 1. Find the equation of a line with slope 2 and y-intercept -3.
- **2.** What is the equation of the y-axis?
- **3.** What is the equation of the line that passes through points (3, 2) and (2, 1)?
- 4. A line l has slope  $\frac{3}{5}$ . What is the slope of a line parallel to l? What is the slope of a line perpendicular to l?
- 5. What is the equation of a line with slope m = 2 and containing the point (1,0)?
- 6. Find k if (1,9) is on the graph of y 2x = k.
- 7. Find k if (1, k) is on the graph of 5x + 4y 1 = 0
- 8. Show that the quadrilateral with the vertices (-1,-2), (4, -1), (5,4), (0,3) is a rhombus. Show that the diagonals are perpendicular.
- **9.** The vertices of a triangle are A(4,3), B(6,-1), C(-2,-5). L,M are midpoints of BC and CA. Find the coordinates of L and M and show that  $LM = \frac{1}{2}BA$
- \*10. A hiker climbs a hill. He starts at 9am and reaches the summit at 4pm. The next day, he returns and starts again at 9am and reaches the base of the mountain at 4pm. Show that there exists a point on the hill where he stood at exactly the same time each day.