MATH 6
HANDOUT 7: SETS CONTINUED

New material introduced today:

We say that set A is a *subset* of B (notation: $A \subseteq B$) if every element of A is also an element of B: $x \in A \Rightarrow x \in B$. Note that A can be equal to B.

1. Let $A = \{1, 2, 3\}$ and $B = \{1, 2, 3, 4\}$. Is it true that $A \subseteq B$?
   Can you find an element of B that is not an element of A?

2. Let
   A=set of all people who know French
   B=set of all people who know German
   C=set of all people who know Russian
   Describe in words the following sets:
   (a) $A \cap B$ (b)$A \cup (B \cap C)$ (c) $(A \cap B) \cup (A \cap C)$ (d) $C \cup \overline{A}$.

3. Let us take the usual deck of cards. As you know, there are 4 suits, hearts, diamonds, spades and clubs, 13 cards in each suit.
   Denote:
   H=set of all hearts cards
   Q=set of all queens
   R=set of all red cards
   Describe by formulas (such as $H \cap Q$) the following sets:
   all red queens
   all black cards
   all cards that are either hearts or a queen
   all cards other than red queens
   How many cards are there in each set?

4. In a class of 25 students, 10 students know French, 5 students know Russian, and 12 know neither. How many students know both Russian and French?

5. For each of the sets below, draw it on the number line and then describe its complement:
   (a) $[0, 2]$ (b) $(-\infty, 1] \cup [3, \infty)$ (c) $(0, 5) \cup (2, \infty)$ where
   $[a, b] = \{x \mid a \leq x \leq b\}$ is the interval from a to b (including endpoints),
   $(a, b) = \{x \mid a < x < b\}$ is the interval from a to b (**not** including endpoints),
   $[a, \infty) = \{x \mid a \leq x\}$ is the half-line from a to infinity (including a),
   $(a, \infty) = \{x \mid a < x\}$ is the half-line from a to infinity (**not** including a)

*6. A barber in a small town decides that he will shave all men who do not shave themselves (and only them). Should he shave himself? [Of course, the barber is a man.]

7. Consider the following sets:
   $\mathbb{Z}$ — all whole numbers (positive and negative)
   $\mathbb{N}$ — all positive whole numbers
   $\mathbb{R}$ — all numbers
   $\mathbb{Q}$ — all rational numbers (i.e., those that can be written as a fraction)
   Order them from smallest to largest, so that each set is a subset of the next one.

8. List all subsets of the set $S = \{1, 2, 3\}$ (do not forget the empty set $\varnothing$ and S itself). Can you guess the general rule: if set S has n elements, how many subsets does it have?

9. Find sets A, B C if you now that $A \cup B = \{1, 3, 4, 5, 7\}$, $B \cup C = \{1, 2, 4, 5, 6, 8, 9\}$, $(A \cup B) \cap C = \emptyset$, $(B \cup C) \cap A = \{1, 5\}$.

10. Find A if you know that $A \cup \{5, 7\} = \{3, 5, 7, 8\}$, $A \cap \{1, 2, 5, 7\} = \{5, 7\}$