MATH 6
HANDOUT 4: LOGIC LAWS

Logic Laws

The logic operations — NOT, AND, OR, XOR — obey certain laws, similar to the usual associativity, distributivity, etc. laws for addition and multiplication. Here are some of the most important laws:

$$\text{NOT}(A \text{ AND } B) = (\text{NOT } A) \text{ OR}(\text{NOT } B)$$
$$\text{NOT}(A \text{ OR } B) = (\text{NOT } A) \text{ AND}(\text{NOT } B)$$
$$A \text{ AND}(B \text{ OR } C) = (A \text{ AND } B) \text{ OR}(A \text{ AND } C).$$

If

Today, we discussed one more logical operation: $\implies$ (reads "implies"), or "if A then B", described by the following truth table:

A	B	$A \implies B$
T	T	T
T	F	F
F	T	T
F	F	T

Homework

When doing the homework, remember to write arguments, not just answers!

As before, some of the questions of this assignment take place on the island of Knights and Knaves, described in the previous homework.

1. What is the LCM and GCD for 8, 40 and 120?

2. What is the LCM and GCD for 26 and 1001?

3. Solve the equation $20 - 3(x - 1) - (x + 1) = 6$

4. What is $(2 \times 5)^3$, $\dfrac{6^5}{2^4}$, $2^5 \times 2^2$

5. Write as a rational the following repeating decimal 6.235235235....

6. An inhabitant, Carl, of the island Knights and Knaves tells you "If Sue is a knave, then this road leads to the capital." However, later you learn that Carl is a knave. What does it tell you?

7. Many trucks carry the message: "If you do not see my mirrors, then I do not see you". Can you rewrite it in an equivalent form without using the word "not"?

8. On the island of Knights and Knaves, a traveler meets two inhabitants: Carl and Bill. Bill says: "Carl is a Knave". Carl says: "If Bill is a Knight, then I am a Knight, too." Can you determine whether each of them is a Knight or a Knave?

9. Check whether $A \implies B$ and $B \implies A$ are equivalent, by writing the truth table for each of them.

10. Check that $A \implies B$ is equivalent to $(\text{NOT } A) \text{ OR } B$ (thus, "if you do not clean up your room, you will be punished" and "clean up your room, or you will be punished" are the same).

11. A mom tells the son "If you do not do the dishes, you will not go to the movie". Is it the same as "If you do the dishes, you go to the movie?"

12. Write the truth table for each of the following formulas. Are they equivalent (i.e., do they always give the same value)?
    (a) $(A \text{ AND } B) \text{ OR} (A \text{ AND } C)$
    (b) $A \text{ AND} (B \text{ OR } C)$.

13. Let us consider a new logical operation, called NAND, which is defined by the following truth table:

A	B	A NAND B
T	T	F
T	F	T
F	T	T
F	F	T

    (a) Show that A NAND B is equivalent to NOT$(A \text{ AND } B)$ (this explains the name: NAND is short for "not and").
    (b) Show that A NAND A is equivalent to NOT A.
    (c) Write the truth table for $(A \text{ NAND } B) \text{ NAND} (A \text{ NAND } B)$.
    (d) Write the truth table for $(A \text{ NAND } A) \text{ NAND} (B \text{ NAND } B)$.
    (e) Show that any logical formula which can be written using AND, OR, NOT can also be written using only NAND.