

Math 6a/d: Homework 17

Deadline: Wednesday, February 17, 2021

Odds and Evens

In the problems that follow you are welcome to use any of the definitions of odd and even numbers that we came up with in class. Remember that different problems that involve odds and evens are easier to solve using different definition.

We defined an even number as:

- A number that ends with 0, 2, 4, 6, 8.
- A number that is divisible by 2.
- A number that can be divided into two equal parts without a leftover (more mathematical term here is a remainder).
- A number that splits into pairs without a leftover.

Similarly, an odd number can be defined as:

- A number that ends with 1, 3, 5, 7, 9.
- A number that is not divisible by 2.
- A number that can be divided into two equal parts with a leftover 1.
- A number that splits into pairs with a leftover 1.

An unknown even number n can be expressed as $2 \times k$, where k is another unknown integer. The notations $n=2 \times k$ makes it clear that the number in question is even.

Similarly, an unknown odd number n can be represented as $2 \times k + 1$.

Definition: **Parity** is the property of a number being either even or odd.

For example, it is easier to say “The parity of the number of upside-down cups does not change” than “If the number of upside-down cups is even, it stays even; if this number is odd, it remains odd.”

Parity of Sum, Difference and Product

$$\text{even} + \text{even} = \text{even} \qquad \text{even} - \text{even} = \text{even}$$

$$\text{odd} + \text{odd} = \text{even} \qquad \text{odd} - \text{odd} = \text{even}$$

$$\text{odd} + \text{even} = \text{odd} \qquad \text{odd} - \text{even} = \text{odd}$$

$$\text{even} + \text{odd} = \text{odd} \qquad \text{even} - \text{odd} = \text{odd}$$

$$\text{even} \times \text{even} = \text{even} \qquad \text{odd} \times \text{even} = \text{even} \qquad \text{odd} \times \text{odd} = \text{odd}$$

Problems

1. Savir the Junior Hacker reprogrammed the elevator in the 100-story Boogle Corporation building: only two buttons are currently working. The first button sends the elevator 8 floors up, and the second one 6 floors down. (The elevator will not move if it is asked to go above the 100th floor or below the 1st floor.)

(a) The company's CEO is currently drinking coffee on the first floor. (There is no lobby floor in the building). Can he take the elevator to the 95th floor? If so, show how. If not, explain why.

(b) Can he take the elevator to the 96th floor? If so, show how. If not, explain why.

2. In the Land of Not So Far Away there live 9 happy and 9 unhappy princesses. Shmerlin the Magician has just learned three new spells. The first spell makes any two unhappy princesses of his choice happy. His second spell transforms any pair of happy princesses into unhappy ones. The third spell switches the moods of a happy princess and an unhappy one: the happy princess becomes unhappy, and the unhappy one becomes happy.

Shmerlin would like to make all the princesses happy. Prove that these three spells are not sufficient for his plan to come true by following these four steps:

(a) Explain what effect each of Shmerlin's spell has on the number of unhappy princesses.

(b) Currently, the number of unhappy princesses is odd. Suppose that Schmerlin utters one of his spells. Prove that the number of unhappy princesses will remain odd.

(c) Suppose that Schmerlin performs several spells in a row. Prove that the number of unhappy princesses will remain odd.

(d) Is it possible for the number of unhappy princesses to eventually go down to zero?

2. Two mountain trolls, Clarence and Terrence, were born exactly one year apart. Today they are celebrating their common birthday. Clarence claims that their total age is 1128 years. However, Michael the hobbit is pretty sure that Clarence is mistaken. How does he know?

3. Can a 5×5 chessboard be tiled by 2×1 dominoes? (to tile means to cover the boards completely, without any overlaps.)

4. While visiting the Knights and Knaves Island, you meet an islander. He says "Yesterday I baked 39 cookie for my party. Every adult at the party ate exactly 2 cookies, and every child ate 4 cookies. The cookies were a big success - by the end

of the evening they'd all been eaten by the guests!" Is this islander a knight or a knave?

5. Slimy the Bog Witch invited the same number of witches and wizards to her birthday party. She bought 115 live frogs as party favors for her guests. She wants each guest to receive the same number of frogs. However, the witch's wise owl tells her that it is not possible to share the frogs this way. How does the owl know?

6. While visiting the Knights and Knaves Island, Kevin the SchoolNova student met an islander. Kevin asked the islander how old he was. The islander replied, "If you multiply my age by 18, you get 1421." Kevin immediately recognized that the islander was a liar - why?

7. Captain Book keep a journal of his adventures. Once, on his way to the Cook Islands, he wrote, "Our flotilla has five ships. Each ship has an odd number of sailors on it. The total number of sailors in our expedition is 5000." Mr. Hobbs, the history professor, claims that can't be right. Why?

8. In the morning there were 5 space jets at the space port of the planet Pandora. During the day, several more jets landed, and a few jets departed from the port. The jets always arrived in groups of 2 or 4, and departed in groups of 2. In the evening, the air dispatcher counted 60 jets at the space port. Prove that the dispatcher miscalculated the number of jets.

9. Shmerlin the Magician inherited a beautiful old book of spells. The book is bound in soft leather, and all of its page numbers are painted in gold. Unfortunately after Shmerlin opened the old book, several loose pages fell out of it. After Shmerlin collected all the loose pages (25 two-sided sheets total), he decided to give himself some practice in math magic: he conjured a spell to add together all the page numbers on all the loose pages. The spell resulted in number 2000. Shmerlin's wise owl claims that Shmerlin conjured an incorrect spell. How does the owl know?

10. The integers from 1 to 22 are written on the board in a row. Can you insert plus and minus signs between them in such a way as to get an expression that is equal to 0?