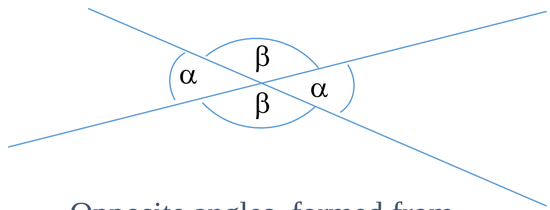
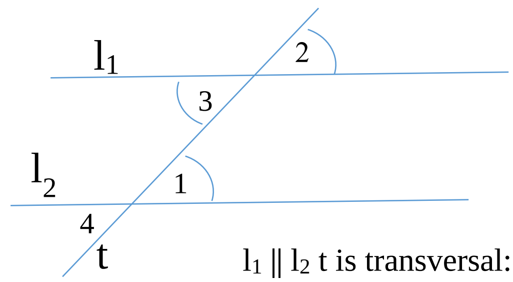
Math 5B: Classwork 22
Homework #22 is due March, 28

Geometry: parallel lines, parallelogram



Opposite angles, formed from crossing straight lines, are equal.



$l_1 \parallel l_2$ t is transversal:
$\angle 1 = \angle 2 = \angle 3$

$\angle\alpha = \angle\alpha$ – opposite

$\angle\alpha + \angle\beta = 180^0$ – on a straight line,

Or complementary angles

$\angle 1 = \angle 3$ = alternate interior angles

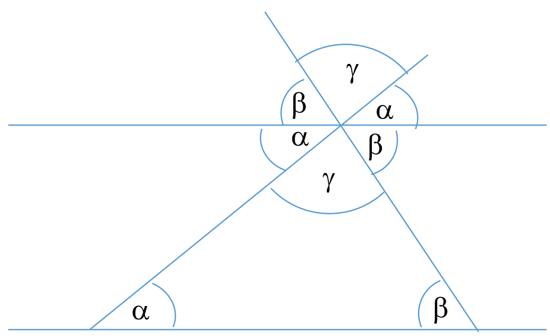
$\angle 1 = \angle 2$ = corresponding angles

$\angle 4 = \angle 2$ = alternate exterior angles

Theorem 1: If two parallel lines (l1 and l2) are intersected by a third line (t), then the formed alternate interior angles are equal.

Theorem 2: If two alternate interior angles formed when two lines (l1 and l2) crossed by a third (t) are equal ($\angle 1 = \angle 3$), then the two lines (l1 and l2) are parallel.

Theorem 3: Show that the sum of angles in a triangle is always 180^o.



Sum of angles of an n-gon

Recall that sum of angles of a triangle is 180^0. Since a quadrilateral can be cut into 2 triangles, sum of angles of a quadrilateral is $2\times180^0 = 360^0$. Similarly, for a pentagon we get 3×180^0, and for an n-gon, the sum of angles is $(n - 2) \times 180^0$.

Congruence

In general, two figures are called congruent if they have the same shape and size. We use the symbol ≅ to denote congruent figures: to say that M_1 is congruent to M_2, one writes $M_1 \cong M_2$.

The precise definition of what the "same shape and size" means depends on the figure. Most importantly, for triangles it means that corresponding sides are equal and corresponding angles are equal: $\Delta ABC \cong \Delta A'B'C'$ is the same as:

$$AB = A'B', \quad BC = B'C', \quad AC = A'C',$$

$$\angle A = \angle A', \quad \angle B = \angle B', \quad \angle C = \angle C'.$$

Note that for triangles, the notation $\Delta ABC \cong \Delta A'B'C'$ not only tells that these two triangles are congruent, but also shows which vertex of the first triangle corresponds to which vertex of the second one. For example, $\Delta ABC \cong \Delta PQR$ is not the same as $\Delta ABC \cong \Delta QPR$.

Congruence tests for triangles

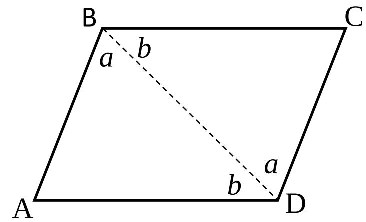
By definition, to check that two triangles are congruent, we need to check that corresponding angles are equal and corresponding sides are equal; thus, we need to check 6 equalities. However, it turns out that in fact, we can do with fewer checks.

Rule 1 (Side-Side-Side rule). If $AB \cong A'B'$, $BC \cong B'C'$ and $AC \cong A'C'$ then $\Delta ABC \cong \Delta A'B'C'$.

This rule is commonly referred to as the $\boldsymbol{SSS}$ rule. This rule – and congruent triangles in general – are very useful for proving various properties of geometric figures. As an illustration, we prove the following useful result.

Theorem. Let $ABCD$ be a quadrilateral in which opposite sides are equal: $AB = CD$, $AD = BC$. Then $ABCD$ is a parallelogram.

Proof. Let us draw a diagonal BD. Then triangles ΔABD and ΔCDB are congruent by $\boldsymbol{SSS}$; thus, the two angles labeled by letter a in the figure are equal; also, the two angles labeled by letter b are also equal. Thus, lines BC and AD are parallel (alternate interior angles!). In the same way we can show that lines AB and CD are parallel. Thus, $ABCD$ is a parallelogram. (It is also true in the opposite direction: in a parallelogram, opposite sides are equal. We will prove it next time.)



One can also try other ways to define a triangle by three pieces of information, such as two sides and an angle between them.

Rule 2 (Angle-Side-Angle Rule). If $\angle$ A = $\angle$ A', $\angle$ B = $\angle$ B' and AB = $A'B'$, then ΔABC≅ $\Delta A'B'C'$. This rule is commonly referred to as ASA rule.

Rule 3 (SAS Rule). If AB = $A'B'$, AC = $A'C'$ and $\angle$ A = $\angle$ A', then ΔABC≅ $\Delta A'B'C'$.

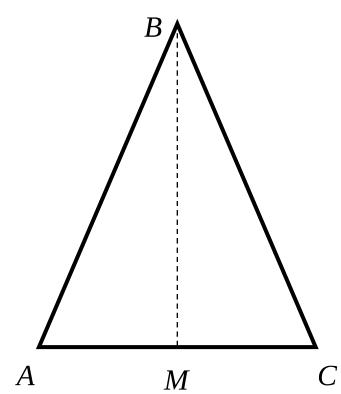
Problem 1: Let ABC be a triangle in which two sides are equal: $AB = BC$ (isosceles triangle). Prove that if M is the midpoint of the side AB, i.e. $AM = MB$, then

- ✓ triangles AMC and BMC are congruent.
- ✓ angles A and B are equal
- ✓ angle $AMC = 90$ degrees

Homework

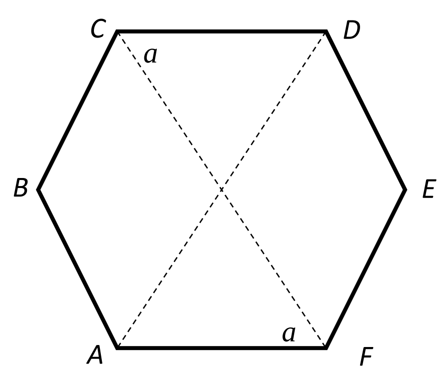
1. Let ABC be a triangle in which two sides are equal: $AB = BC$ (isosceles triangle). We proved in class that if M is the midpoint of the side AB, i.e. $AM = MB$, then
   - ✓ triangles AMC and BMC are congruent.
   - ✓ angles A and B are equal
   - ✓ angle $AMC = 90$ degrees

   So, in an isosceles triangle the median is also a height.
   Please, review your notes and prove the above 3 points again!



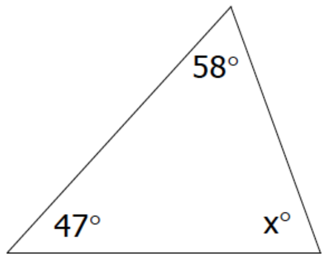
2. Can you prove the following: If in the triangle ΔABC $\angle A = \angle C$ and the point M of the side AC is such that BM is a height (angle $AMB = 90^o$) then this triangle is isosceles?

3. An *n-gon* is called *regular* if all sides are equal and all angles are also equal.
   (a) How large is each angle in a regular hexagon (6-*gon*)?
   (b) Show that in a regular hexagon, opposite sides are parallel. (This is the reason why this shape is used for nuts and bolts).
   [Hint: Show that ACDF is a rectangle, OR show that each of the angles labeled by the letter *a* in the figure is equal to 60^o, and then use the theorem about alternate interior angles.



4. Find the value of x. Show ALL steps!

a)



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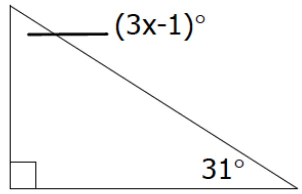

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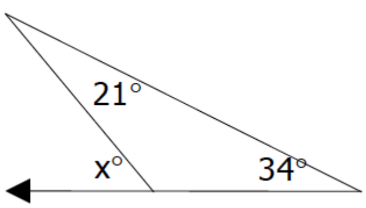

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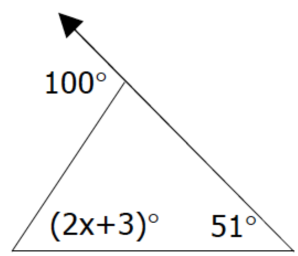

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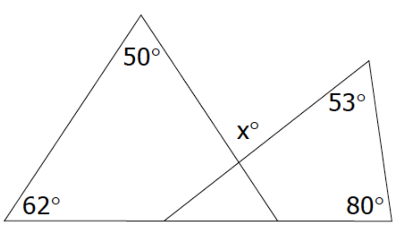

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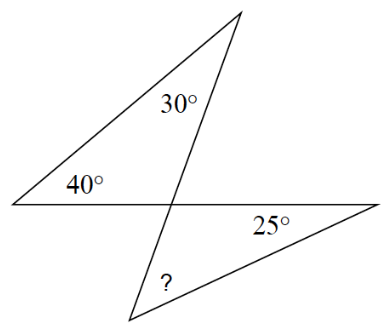
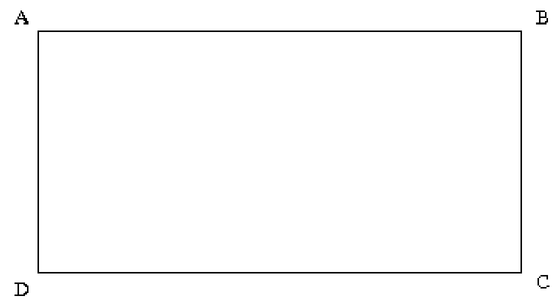


5. Victoria walks along the edges of a rectangular pool from point A to B to C to D, a distance of 38 meters. Julia walks along the edges of the same pool from B to C to D to A, distance of 31 meters. What is the perimeter of the pool in meters?



6. The six-digit number $63X904$ is an even multiple of 27. What digit does X represent?

7. Simplify the expressions:
(a) $x-(1+5x)=$
(b) $2x-(3x^2+x-1)+(2+2x-x^2)=$
(c) $3x(-2xy)=$
(d) $(y-5)(y-1)-(y+2)(y-3)=$
(e) $3(x-1)^2-3x(x-5)=$

8. The average weight of a group of children is 100 pounds. Todd, who weighs 112 pounds, then joins the group. This raises the average weight of the group to 102 pounds. How many children were in the original group?

9. * Problem we discussed in class. On the table in front of Avya there are 1000 quarters, 990 tails and 10 heads. Avya is blindfolded and cannot tell the difference between a head and a tail. How can Avya make sure that if she splits the coins in 2 groups, the number of heads in each group is the same?

   Hints:

   - The groups do not have to be equal;
   - Even though Avya is blindfolded, she can count how many coins are in each group.
   - After Avya divides coins into two groups she needs to perform one operation on one of the groups. And someone in class already said what that operation should be.