Math 5B: Classwork 20
Homework #20 is due March 14-th.

1. Solve the following equations for *(don't forget to check your solutions):*

   (a) $\frac{28x-14}{20-10x} + 5x = 4$

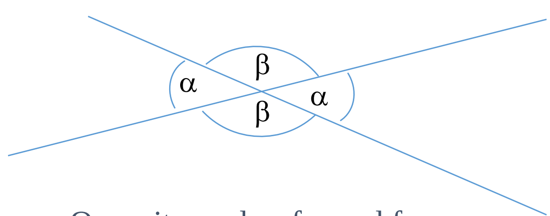
   (b) $\frac{10y+30a}{60a-20y} - 6 = -1, find\ y$

2. Solve the equation

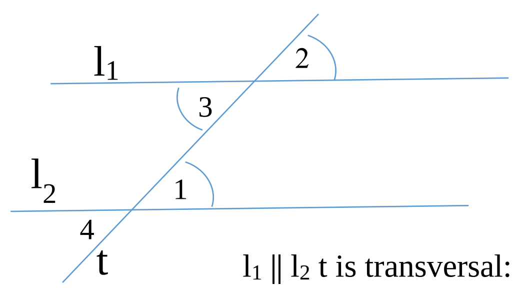
$$(9-2x)^2 - (2x-7)(2x+7) = 202$$

3. Using the same sequence of steps we did in class, perform division of 80/7. Show M, N, all Ai (quotient) and Ri (remainders). How many "pigeonholes" do we have in this case and what goes into those pigeonholes?

# *Geometry: parallel lines, parallelogram*




Opposite angles, formed from crossing straight lines, are equal.




$l_1 \parallel l_2$ t is transversal:
$\angle 1 = \angle 2 = \angle 3$

$\angle\alpha = \angle\alpha$ – opposite

$\angle\alpha + \angle\beta = 180^0$ – on a straight line,

Or complementary angles

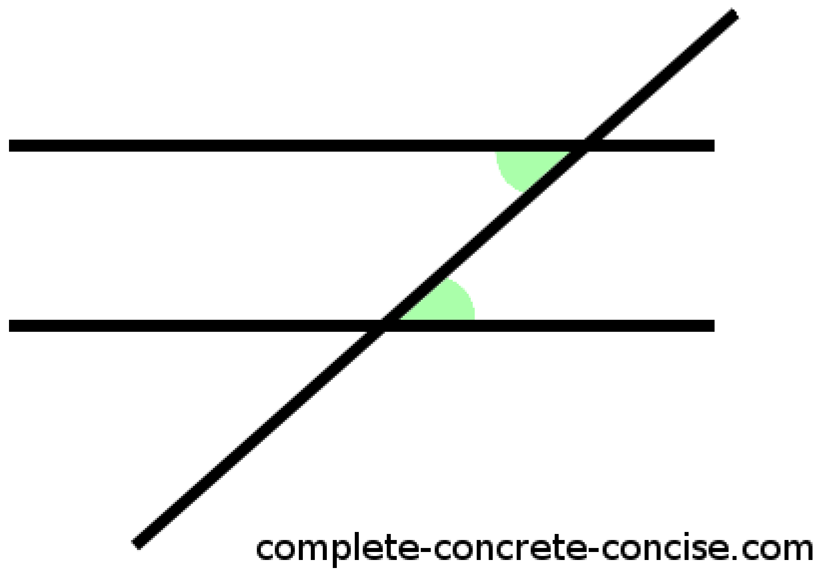
$\angle 1 = \angle 3$ = alternate interior angles

$\angle 1 = \angle 2$ = corresponding angles

$\angle 4 = \angle 2$ = alternate exterior angles

**Theorem 1:** If two parallel lines (l1 and l2) are intersected by a third line (t), then the formed alternate interior angles are equal.

This is illustrated in the image below:




We see two parallel lines and a third line (transversal) intersecting (crossing or cutting through) both of them. The green shaded angles are: (1) inside (between) the two parallel lines, (2) congruent (identical or the same), and (3) on opposite sides of the transversal.

This is true for the other two unshaded interior angles. It is also true for the alternate exterior angles (but not proved here).

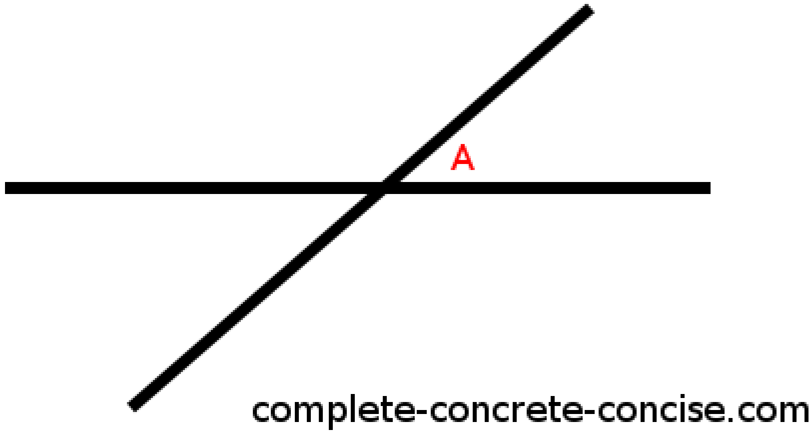
**Axioms**

Proofs are built on two things: (1) postulates, axioms, or hypotheses – these are things that are assumed to be true, but can't be proven. In general, we try to use the fewest number of axioms we can; (2) other proofs.

This proof depends on two axioms: (1) if you pick any two distinct points on a straight line, the angle between those two points will be 180°; (2) if you take any two intersecting straight lines and shift one of the lines so it it is in a different position, but still parallel to its original position, the angle between the two intersecting lines stays the same.

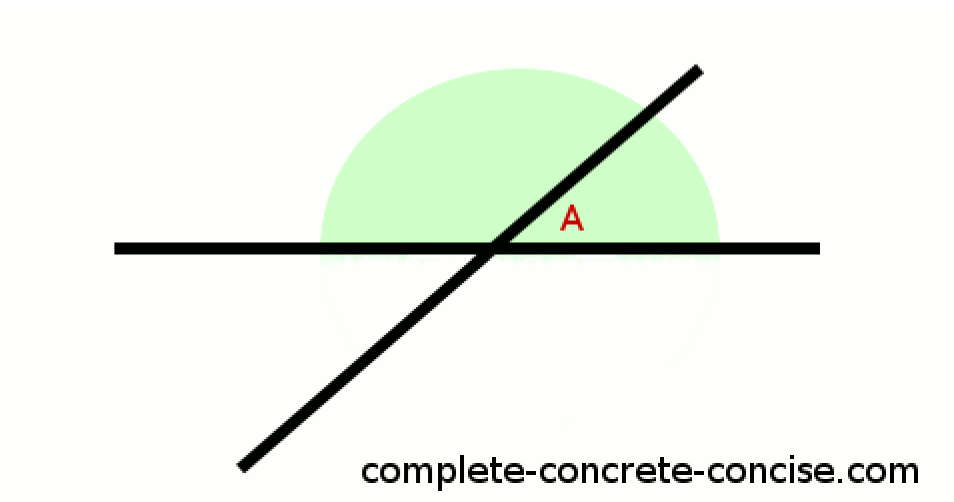
Proof

1) Consider two intersecting lines:

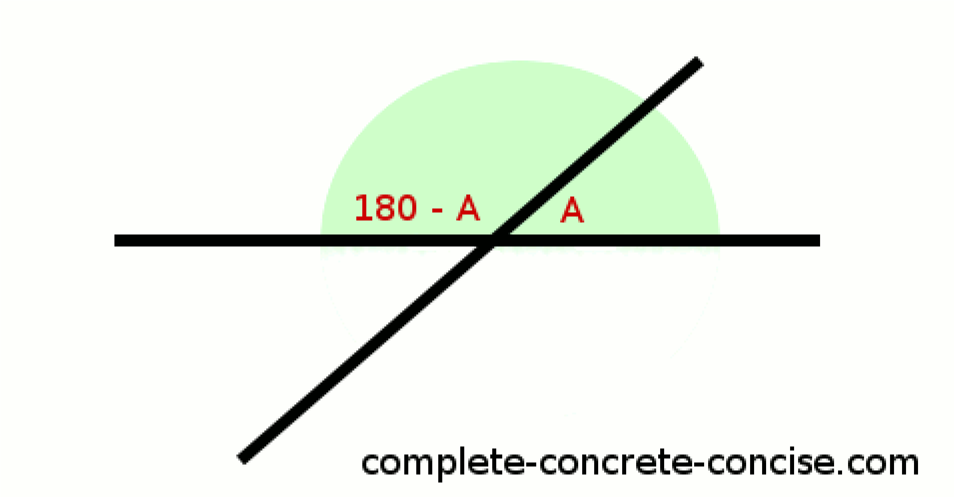



At the point of intersection, there is an angle (called A).

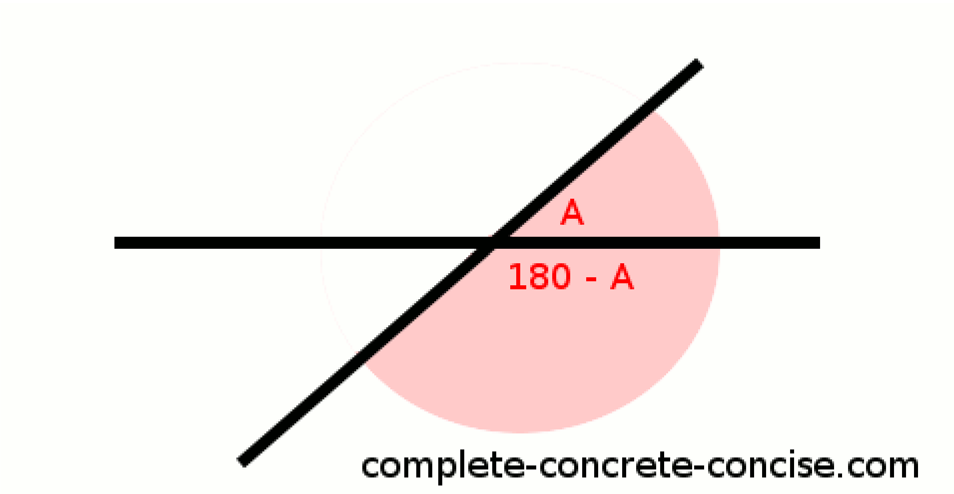
2) If the line is straight, the angle between any two points on that line must be 180° (axiom #1 from above):



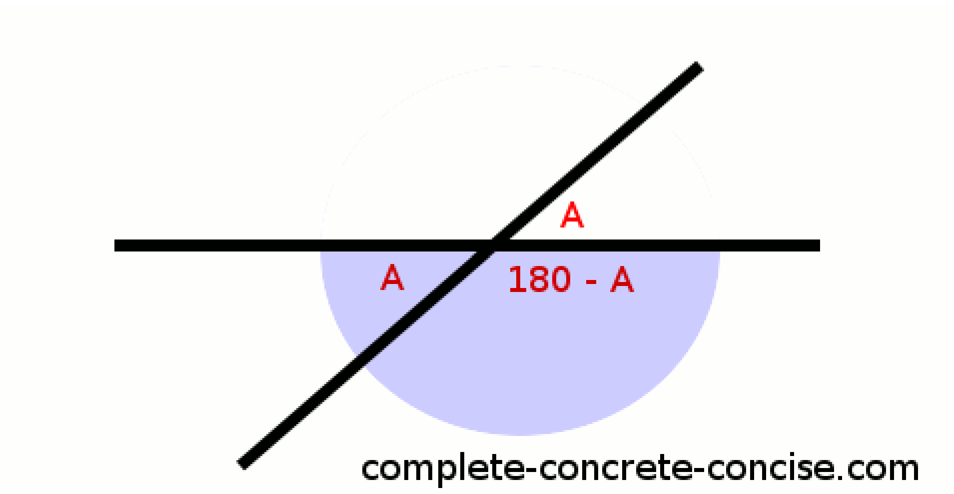

3) If one angle is A degrees, then the other angle must be 180° – A:




4) This is true for both straight lines:



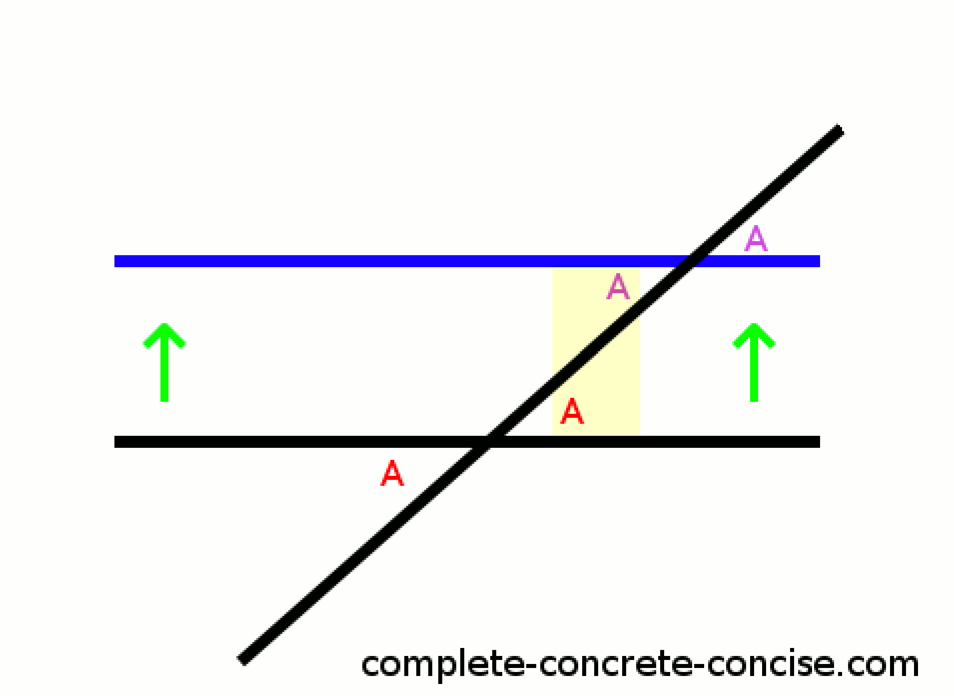

5) This means the remaining angle must be 180° – (180° – A) = A degrees:



This proves that angles on alternate sides of the transversal (at the point of intersection) are congruent (identical).

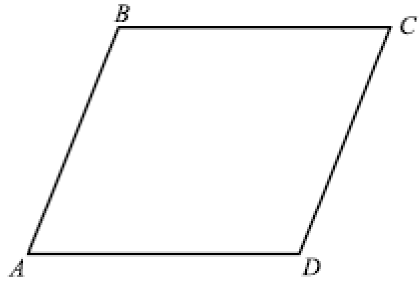
6) If we draw a line parallel to one of the lines (it doesn't matter which) and it intersects the other line (that line is now called a transversal), we know that the angles of intersection must be the same (axiom #2 from above).

If the angles of intersection are the same for both lines, then the alternate interior angles must be the same:




**Theorem 2:** If two alternate interior angles formed when two lines (l1 and l1) crossed by a third (t) are equal ($\angle 1 = \angle 3$), then the two lines (l1 and l2) are parallel. (<u>added to the homework</u>).

**Parallelogram:** A parallelogram is a quadrilateral in which opposite sides are parallel. Parallelograms have a number of interesting properties, which we will study later.




**Sum of angles of an n-gon**

Recall that sum of angles of a triangle is $180^0$. Since a quadrilateral can be cut into 2 triangles, sum of angles of a quadrilateral is $2\times180^0 = 360^0$. Similarly, for a pentagon we get $3\times180^0$, and for an n-gon, the sum of angles is $(n - 2) \times 180^0$.

# ***Classwork / Homework***

1. Is it true that any rectangle is also a parallelogram? Is it true that any parallelogram is a rectangle? Try to argue as carefully as you can.

   **<u>Theorem (Pythagorean theorem).</u>** In a right triangle with legs *a*, *b* and hypotenuse *c*, one has:

$$a^2 + b^2 = c^2$$

$$c = \sqrt{a^2 + b^2}$$

   The area of the triangle is **half of the base times height**.

   - "b" is the distance along the base
   - "h" is the height (measured at right angles to the base)

   Area = ½ × b × h

2. Can one cut from the circle of diameter 9.9 m?

   *a)* A rectangle with the sides of 6m and 8m;

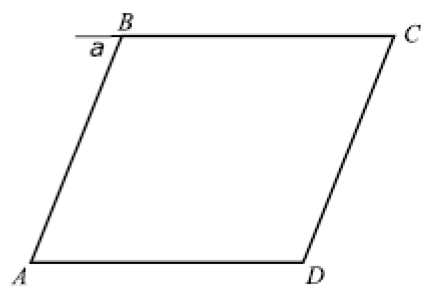
   *b)* A square with the side of 7m;

   *Please provide the step by step solution similar to how we did it in class.*

3. *T*he area of an isosceles (2 equal sides -legs) triangle is 60 square meters. The height drawn to the base (third side) is 5m. *The height of an isosceles triangle divides its base side in half. Find all sides of the triangle. Please write out the full solution similar to how it was done in class.*

4. Theorem 2: If two alternate interior angles formed when two lines (l1 and l1) crossed by a third (t) are equal ($\angle 1 = \angle 3$), then the two lines (l1 and l2) are parallel.

5. Show that in a parallelogram, diagonally opposite angles are equal $\angle A = \angle C$, $\angle B = \angle D$ [Hint: see figure below, we did this in class, try to repeat the arguments]




6. Show that the previous problem also works in the other direction: if in a quadrilateral, diagonally opposite angles are equal: $\angle A = \angle C$, $\angle B = \angle D$, then the quadrilateral must be a parallelogram.

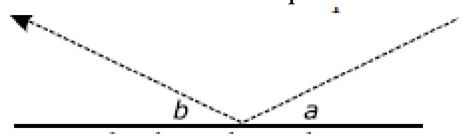
7. Cut two identical paper triangles (the easiest way to do it is to fold a sheet of paper in two and then cut). Can you put these two triangles together so that they form a parallelogram? Will your method always work? Why?

8. An n-gon is called regular if all sides are equal and all angles are also equal.
   (a) How large is each angle in a regular hexagon (6-gon)?
   (b) How large is each angle in a regular heptagon (7-gon)?

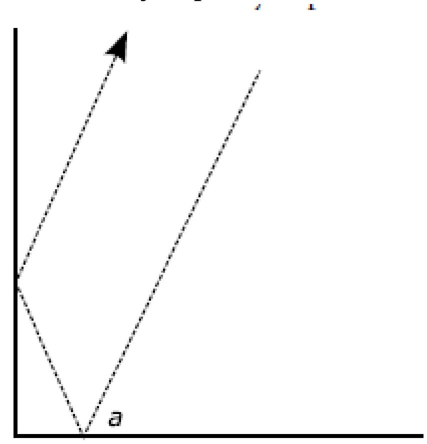
9. Antonia and Gabi took a 9-mile trip in a rowboat. There was only one pair of oars, so they took turns rowing (however, they didn't time how long each of them was rowing, so it could happen that one had rowed longer than the other). Antonia could row at the speed of 3 miles per hour; Gabi could only do 2 miles per hour. It took them 3.5 hours to complete the trip. Can you find out how long each of them was rowing?

10. Using the same sequence of steps we did in class, perform division of **200**/7. Show M, N, all Ai (quotient) and Ri (remainders). How many "pigeonholes" do we have in this case and what goes into those pigeonholes?

11. * The reflection law states that the angles formed by the incoming light ray and the reflected one with the surface of the mirror are equal: $\angle a = \angle b$




Using this law, show that a corner made of two perpendicular mirrors will reflect any light ray exactly back: the reflected ray is parallel to the incoming one.




[Hint: find the angle which each of these lines form with the horizontal]
This property – or rather, similar property of corners in three dimensions — is widely used: reflecting road signs, tail lights of a car, reflecting strips on clothing are all constructed out of many small reflecting corners so that they reflect the light of a car headlamp exactly back to the car.

12.* Problem we discussed in class. On the table in front of Avya there are 1000 quarters, 990 tails and 10 heads. Avya is blindfolded and cannot tell the difference between a head and a tail. How can Avya make sure that if she splits the coins in 2 groups, the number of heads in each group is the same?

Hints: 1) The groups do not have to be equal;
2) Even though Avya is blindfolded, she can count how many coins are in each group.
3) After Avya divides coins into two groups she needs to perform one operation on one of the groups. And someone in class already said what that operation should be.