Math 5B: Classwork 18
Homework #18 is due February 28-th

Rational numbers

Definition: A **rational** number is a number that can be in the form p/q where p and q are integers and q is not equal to zero.

Example: 2/3 is a rational number because 3 and 2 are both integers

Pigeonhole principle states that if ***n*** items are put **into m** **pigeonholes** (containers) with ***n* > *m***, then at least one pigeonhole (container) must contain more than one item.

In layman's terms, if you have more "objects" than you have "holes," at least one hole must have multiple objects in it.

Theorem: **Any rational number is a finite or repeating decimal**. We prove it using the **Pigeonhole principle**.

Example of the proof is provided in Problem #8 here -> https://www.math.ksu.edu/~zlin/m510/hw1.pdf

Proof. Assume that n is a positive integer. Apply the division algorithm to get m = q*n + r_0 where m is the dividend, n is the divisor, q is the quotient (integer part of the rational number m/n), and r_0 is the remainder with 0 ≤ r_0 ≤ n−1. To compute the tenths place digit a_1, one uses the division algorithm r_0 * 10 = a_1 * n + r_1 where 0 ≤ r_1 ≤ n−1. More generally, to get the 10^{-i} th place digit a_i, one uses the remainder r_{i-1} and division algorithm r_{i-1} × 10 = a_i * n + r_i. When i = n, we have n+1 remainders with the values ranging from 0 to n−1. Thus, by the pigeonhole principle, there must be two equal remainders, say, r_i = r_j where i < j. Let j be the smallest possible number such that r_j = r_i for some i < j. Then a_{j+1} = a_{i+1} which translates into r_{j+1} = r_{i+1} and, correspondingly, a_{j+k} = a_{i+k} with r_{j+k} = r_{i+k}. This shows that the decimal is repeating with the repeating part $a_i a_{i+1} a_{i+2} \ldots a_{j-1}$.

Review

1. Operations with powers:

$$a^n = a \cdot a \cdot \cdots \cdot a\ (ntimes)$$

$$(a \cdot b)^n = a^n \cdot b^n$$

$$a^m \cdot a^n = a^{m+n};$$

$$a^m \div a^n = a^{m-n}$$

$$a^0 = 1$$

$$a^{-n} = \frac{1}{a^n}$$

2. We reviewed solving equations and solving rational equations by multiplying both sides of the equation with the denominator, for example.

$$\frac{(x+1)}{3} = 7$$

$$\frac{(x+1)}{3} \times 3 = 7 \times 3$$

$$(x+1) = 21$$

$$x = 20$$

We also revised the ***identities***:

$$(a+b)^2 = a^2 + 2ab + b^2$$

$$(a-b)^2 = a^2 - 2ab + b^2$$

$$(a+b)(a-b) = a^2 - b^2$$

And ***factorizing***:

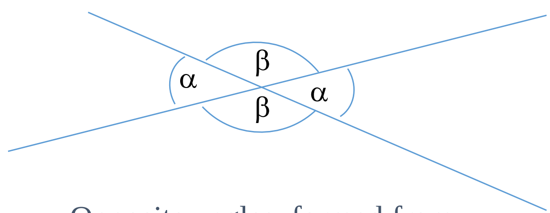
$$a(b+c) = ab + ac$$

… and used them to solve equations.

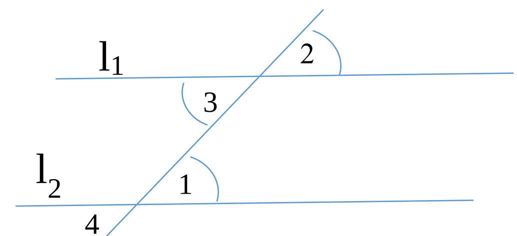
We solved equations with exponents: $a^x = a^c$ and found out that if we have equal bases we need only compare the exponents (powers) to find the unknown: $x = c$.

So, we need to find a way to rewrite the equations where both sides have the same base.

Geometry: Angles



Opposite angles, formed from crossing straight lines, are equal.



t

$l_1 \parallel l_2$ t is transversal:

$\angle 1 = \angle 2 = \angle 3$

$\angle \alpha = \angle \alpha$ – opposite

$\angle 1 = \angle 3$ = alternate interior angles

$\angle \alpha + \angle \beta = 180^0$ – on a straight line, are called supplementary angles

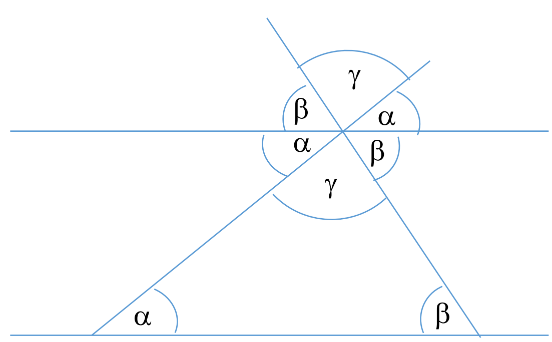
$\angle 1 = \angle 2$ = corresponding angles

Complementary angles

$\angle 4 = \angle 2$ = alternate exterior angles

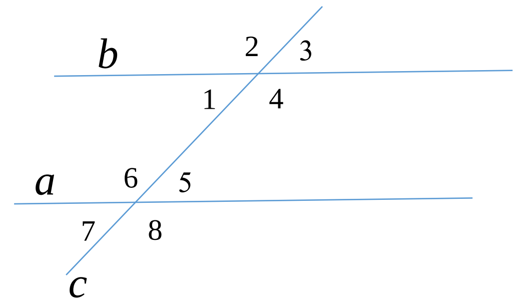
Add up to 90 degrees.

From both these pieces of information we can show that the sum of angles in a triangle is always 180^o.



Homework

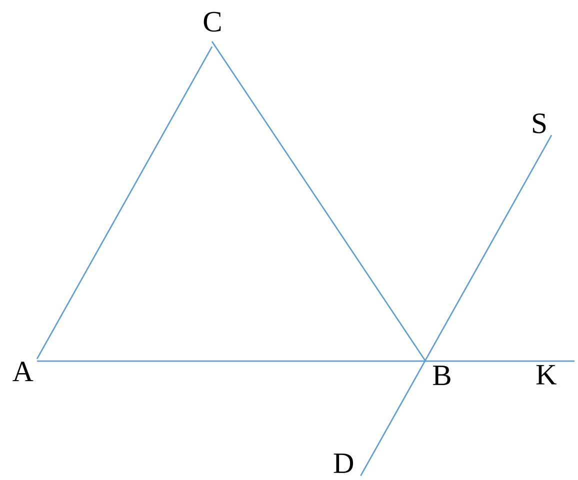
1. On the picture, *a* and *b*, which are parallel to each other, are intersected by line *c*. What are the relationships:
   (a) $\angle 3$ and $\angle 5$
   (b) $\angle 2$ and $\angle 8$
   (c) Prove that $\angle 4 + \angle 5 = 180^o$.



2. In the same picture,
   (a) if $\angle 7 = 65^o$, find: $\angle 1$, $\angle 3$, $\angle 1 + \angle 6$
   (b) If you know that $\angle 7 = \angle 1$, prove that*: $\angle 1 = \angle 3$ and $\angle 5 = \angle 1$

   (* *or say why the angles will be equal*)

3. Intersecting at point B on triangle ABC is drawn line DS, such that DS is parallel to AC. Prove that (or say why the angles will be equal):
   (a) $\angle ACB = \angle SBC$
   (b) $\angle CAB = \angle DBA$
   (c) $\angle CAB = \angle SBK$
   (d) If $\angle CAB = 40^o$ and $\angle BCA = 60^o$, find angles $\angle ABD$ and $\angle SBC$



4. In triangle ABC, $\angle A = 35^o$, $\angle B = 55^o$, prove that this triangle is right-angled.

5. What type of triangle has one angle equal to the sum of the other two?

6. Find each of the outside angles of a right-triangle, if one of its angles is 58^o.

7. Consider the sequence 7, 7^2, 7^3, . . .7^n . . .

   (a) Show that there will be two numbers in this sequence which have the same last two digits. [*Hint: pigeonhole principle!*]

   (b) Show that from some moment, the last two digits of numbers in this sequence will start repeating periodically.

7 ^ 1 = 7

7 ^ 2 = 7 * 7 = 49

7 ^ 3 = 7 * 7 * 7 = ……3 = 49 * 7

7 ^ 4 = 7 * 7 * 7 * 7 = ……. 1 = 49 * 49 = …… 1