Math 5B: Classwork 17
Homework #17 is due February 21-st (HAVE A NICE VACATION!)

Rational numbers

Definition: A **rational** number is a number that can be in the form p/q where p and q are integers and q is not equal to zero.

Example: 2/3 is a rational number because 3 and 2 are both integers

Pigeonhole principle states that if ***n*** items are put **into m** **pigeonholes** (containers) with ***n* > *m***, then at least one pigeonhole (container) must contain more than one item.

In layman's terms, if you have more "objects" than you have "holes," at least one hole must have multiple objects in it.

Theorem: any rational number is a finite or repeating decimal. The way we proved is using **Pigeonhole principle**.

Proof. First we assume that n is a positive integer. Apply the division algorithm to get m = qn + r0 with 0 ≤ r0 ≤ n−1. Here q is the integer part of the rational number m/n. To compute the tenth place digit a1, one uses the division algorithm r0 ×10 = a1n + r1 with 0 ≤ r1 ≤ n−1. More generally, to get the 10−if place digit ai, one uses the remainder ri−1 and division algorithm ri−1×10 = ain+ri. When i = n, then the n+1 remainders r0,r1,...,rn have value ranging from 0 to n−1. Thus, by the pigeonhole principle, there must be two equal remainders, say, ri = rj with i < j. Let j be the smallest possible number such that rj = ri for some i < j. Then aj+1 = ai+1 with ri+1 = rj+1 and, inductively, ai+k = aj+k and ai = ai+(j−i) = ai+2(j−i) = ⋯. This shows that the decimal is repeating with the repeating part aiai+1⋯aj−1.

Review

1. Operations with powers:

$$a^n = a \cdot a \cdot \cdots a \ (n times)$$

$$(a \cdot b)^n = a^n \cdot b^n$$

$$a^m \cdot a^n = a^{m+n};$$

$$a^m \div a^n = a^{m-n}$$

$$a^0 = 1$$

$$a^{-n} = \frac{1}{a^n}$$

2. We reviewed solving equations and solving rational equations by multiplying both sides of the equation with the denominator, for example.

$$\frac{(x+1)}{3} = 7$$

$$\frac{(x+1)}{3} \times 3 = 7 \times 3$$

$$(x+1) = 21$$

$$x = 20$$

We also revised the ***identities***:

$$(a+b)^2 = a^2 + 2ab + b^2$$

$$(a-b)^2 = a^2 - 2ab + b^2$$

$$(a+b)(a-b) = a^2 - b^2$$

And ***factorizing***:

$$a(b+c) = ab + ac$$

… and used them to solve equations.

We solved equations with exponents: $a^x = a^c$ and found out that if we have equal bases we need only compare the exponents (powers) to find the unknown: $x = c$.

So, we need to find a way to rewrite the equations where both sides have the same base.

1. Consider the sequence 7, 7^2, 7^3, . . .7^n (7 to the power of n) where n> 100.

   (a) Show that there will be two numbers in this sequence which have the same last two digits. [*Hint: pigeonhole principle!*]

   (b) Show that from some moment, the last two digits of numbers in this sequence will start repeating periodically.

2. Find *n* for

   (a) $25^{-n} = 5$
   (b) $125^{-n} = {}^{1}\!/_{5}$
   (c) $49^{-n} = 343$

3. Simplify the following expressions:

a) $\frac{3}{X-4} - \frac{5}{X+3} =$

b) $\frac{3(x+2)}{X} + \frac{3-2x}{-X+7} =$

c) $\frac{3x-2}{2x+4} - \frac{7x-6}{x+2} =$

d) $(2 + \frac{4}{x}) : (3x + 6) =$

e) $\left(3 + \frac{3}{x}\right) : \left(3 - \frac{3}{x}\right) =$

f) $\left(5 + \frac{10}{x}\right) : \left(10 - \frac{20}{x}\right) * (x^2 - 4) =$

4. Perform the following arithmetic operations with binary numbers <u>without</u> converting the numbers to decimal form. Then check your answers by converting to decimal and comparing results.

   (a) 100101**b** + 101011**b**

   (b) 10111**b** × 1001**b**

   (c) (110101**b** + 11101**b**) × 10110**b**

5. Base 16 numbers: Perform the following arithmetic operations <u>without</u> converting the numbers to decimal form. Then check your answers by converting to decimal and comparing results.

$$\begin{array}{r} 1F67 \\ \underline{+\ D295} \end{array} \qquad\qquad \begin{array}{r} FD7A \\ \underline{-\ ABCD} \end{array}$$

6. Simplify:

   a) $\sqrt{9} + \sqrt{25} =$

   b) $3\sqrt{9} - 16 =$

c) $0.1\sqrt{400} + 0.2\sqrt{1600} =$

d) $\sqrt{81} - \sqrt{144} =$

e) $-2.7\sqrt{169} - 1.5\sqrt{900} =$

f) $\frac{1}{3}\sqrt{324} + \frac{2}{7}\sqrt{441} =$

7. Simplify:

a) $\frac{\sqrt{999}}{\sqrt{111}} =$

b) $\sqrt{162} \cdot \sqrt{2} =$

c) $\frac{\sqrt{2}}{\sqrt{18}} =$

d) $\sqrt{10} \cdot \sqrt{40} =$

e) $\frac{\sqrt{15}}{\sqrt{735}} =$

f) $\sqrt{\frac{2}{3}} \cdot \sqrt{\frac{3}{8}} =$

8. $$\frac{3.75 \div 1\frac{1}{2} + \left(1.5 \div 3\frac{3}{4}\right) \cdot 2\frac{1}{2} + \left(1\frac{1}{7} - \frac{23}{49}\right) \div \frac{22}{147}}{2 \div 3\frac{1}{5} + \left(3\frac{1}{4} \div 13\right) \div \frac{2}{3} - \left(2\frac{5}{18} - \frac{17}{36}\right) \cdot \frac{18}{65}} =$$

9. Simplify. Write your answer as a number with a positive power:

a) $\frac{4}{9}ab^3 \cdot \frac{3}{2}ab =$

b) $-0.6a^2b \cdot (-10ab^2) =$

c) $ab \cdot (-7ab^3) \cdot 4a^2b =$

d) $(-c^3)^3 \cdot 0.15c^4 =$

e) $10x^2y \cdot (-xy^2) \cdot 0.6x^3 =$

f) $(-2x^3)^2 \cdot \left(\frac{1}{4}x^4\right)^3 =$

10. Solve each equation. Do not forget to perform the check.

a) $|-x+5| = 13$

b) $|-3x - 2(x+5)| = 45$

c) $2\text{x} - \frac{x-6}{5} = 7 + 3\text{x}$

d) $\frac{x}{3} + \frac{x+1}{2} = 2$

Solve the following word problems by writing an equation. Make sure to show all steps!

11. On the first of three tests, Mridula scored 72 points. On the third test, her test score was 1 point more than on the second one. Her average on the three tests was 83. What were her scores on the second and third tests?

12. The difference between two integers is 9. Five times the smaller is 7 more than three times the larger. Find the numbers.

13. The upper Angel Falls, the highest waterfall on Earth, are 750 m higher than Niagara Falls. If each of the falls were 7 m lower, the upper Angel Falls would be 16 times as high as Niagara Falls. How high is each waterfall?

14.* Problem we discussed in class. On the table in front of Avya there are 1000 quarters, 990 tails and 10 heads. Avya is blindfolded and cannot tell the difference between a head and a tail. How can Avya make sure that if she splits the coins in 2 groups, the number of heads in each group is the same?

Hint: 1) The groups do not have to be equal;
2) Even though Avya is blindfolded, she can count how many coins are in each group.
3) After Avya divides coins into two groups she needs to perform one operation on one of the groups.