Math 5b: Classwork 16
Homework #16 is due February 7

DEFINITIONS, THEOREMS, AXIOMS

Definition of Axiom (mathsisfun.com)

Theorem Definition (Illustrated Mathematics Dictionary) (mathsisfun.com)

AXIOMS – a statement that is taken to be true.

THEOREM – a statement that has been proven to be true.

- TO PROOF a theorem need to show the statement is ALWAYS true; to show a theorem is false, need to cite ONLY one example where it's false.

Definition: A **rational** number is a number that can be in the form p/q where p and q integers and q is not equal to zero and the fraction $\frac{p}{q}$ cannot be simplified further. Example: 2/3 is a rational number because 3 and 2 are both integers and the fraction $\frac{p}{q}$ cannot be simplified further.

Theorem: The square-root of 2 ($\sqrt{2}$) is not a rational number, i.e. it cannot be written as a fraction.
Proof: Let us assume that $\sqrt{2} = \frac{p}{q}$ where p and q are some whole numbers and the fraction $\frac{p}{q}$ cannot be simplified further. We can write:

$$\left(\sqrt{2}\right)^2 = \left(\frac{p}{q}\right)^2$$

$$2 = \frac{p^2}{q^2}$$

So that:
$$2q^2 = p^2$$

Thus, p^2 must be an **even** number. That means p must be an **even** number and could be rewritten as: *p = 2m where m is the whole number*. Substituting:

$$2q^2 = p^2 = 4m^2$$

So that:
$$q^2 = 2m^2$$

Thus, q must be an even number. This *contradicts* our initial assertion that $\frac{p}{q}$ could not be simplified further (at least each p, q could be reduced by one factor of 2 each). Therefore, we have proven by contradiction that $\sqrt{2}$ cannot be written as a rational number.

Review

1. Operations with powers:

$$a^n = a \cdot a \cdot \cdots \cdot a \; (n times)$$

$$(a \cdot b)^n = a^n \cdot b^n$$

$$a^m \cdot a^n = a^{m+n};$$

$$a^m \div a^n = a^{m-n}$$

$$a^0 = 1$$

$$a^{-n} = \frac{1}{a^n}$$

2. We reviewed solving equations and solving rational equations by multiplying both sides of the equation with the denominator, for example.

$$\frac{(x+1)}{3} = 7$$

$$\frac{(x+1)}{3} \times 3 = 7 \times 3$$

$$(x+1) = 21$$

$$x = 20$$

We also revised the ***identities***:

$$(a+b)^2 = a^2 + 2ab + b^2$$

$$(a-b)^2 = a^2 - 2ab + b^2$$

$$(a+b)(a-b) = a^2 - b^2$$

And ***factorizing***:

$$a(b+c) = ab + ac$$

… and used them to solve equations.

We solved equations with exponents: $a^x = a^c$ and found out that if we have equal bases we need only compare the exponents (powers) to find the unknown: $x = c$.

So, we need to find a way to rewrite the equations where both sides have the same base.

Homework

1. Solve the following equations for *x*:
   (a) $\frac{5y-12}{3-2y} = 2$

   (b) $\frac{8-2x}{3x-1} = 3$

   (c) $\frac{3x+a}{2a-5x} = -1$

2. Solve the equation:

$$(x-3)^2 - (x-5)(x+5) = 4$$

3. Simplify the fractions using the above identities and factoring rules:
   (a) $\frac{y^2-16}{3y+12} =$

   (b) $\frac{a^2+10a+25}{a^2-25} =$

   (c) $\frac{15z^2-9z}{25z^2-9} =$

4. Simplify the following expressions:
   a) $\frac{2}{X-2} - \frac{5}{X+3} =$

   b) $\frac{3x}{X} - \frac{5x-2}{X+6} =$

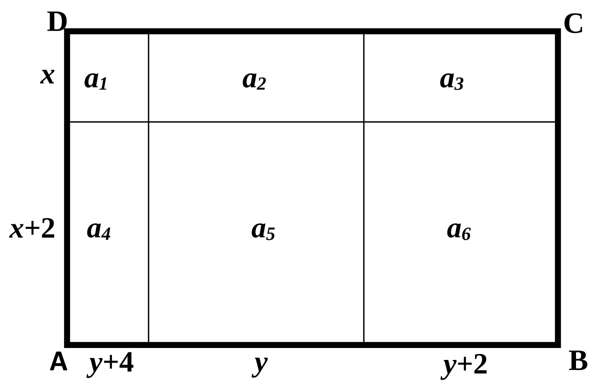
   c) $\frac{3x-2}{X} + \frac{7x-6}{X-8} =$

5. Consider the sequence 7, 72, 73, . . .7n . . .

   (a) Show that there will be two numbers in this sequence which have the same last two digits. [*Hint: pigeonhole principle!*]

   (b) Show that from some moment, the last two digits of numbers in this sequence will start repeating periodically.

6. ABCD, below, is a rectangle which is split into 6 smaller ones by 3 parallel lines. Find:
(a) The area of each rectangle
(b) The sum of the areas of the 6 rectangles
(c) The total area ABCD
(d) Compare (b) and (c)



7. A worker is earning \$24 for each day he works, but he has to pay back \$6 for each day he takes off. After 30 days he ended up receiving no money. How many days did he work?

8. Find *n* for
(a) $3^{-n} = 3$
(b) $3^{-n} = {}^{1}/_{3}$
(c) $9^{-n} = 81$

9. (*from 101 puzzles in thought and logic, by C. R. Wylie*) Clark, Jones, Morgan, and Smith are four men whose occupation are butcher, druggist, grocer, and policeman, though not necessarily in that order.

Clark and Jones are neighbors and take turns driving each other to work.

Jones makes more money than Morgan.

Clark beats Smith regularly at bowling.

The butcher always walks to work.

The policeman doesn't live near the druggist.

The only time the grocer and the policeman ever meet is when the policeman arrested the grocer for speeding.

The policeman makes more money than the druggist or the grocer.

What is each man's occupation?

10.* Problem we discussed in class. On the table in front of Avya there are 1000 quarters, 990 tails and 10 heads. Avya is blindfolded and cannot tell the difference between a head and a tail. Help Avya to split the coins in 2 groups, so that the number of heads in each group is the same.

Hint: 1) The groups do not have to be equal;
2) Even though Avya is blindfolded, she can count how many coins are in each group.

Now that we've refreshed in class how to calculate areas, the following problems become due if not completed before:

[What is Area? (mathsisfun.com)]

Definition and Area of a triangle: [Triangles - Equilateral, Isosceles and Scalene (mathsisfun.com)]

Definition and Area of a trapezoid: [Trapezoid (mathsisfun.com)]

From Homework 15

11 (#3 in HW 15). The side of an equilateral triangle is 1m. Find its height and area. Reminder: an equilateral triangle has all sides the same length.

From Homework 14

12. (#4 in HW 14) Find the height and area of the figure:
Three sides are given and the two marked angles are right angles.