Math 5b: Classwork 15
Homework #15 is due January 31

Definition: A **rational** number is a number that can be in the form p/q where p and q are integers and q is not equal to zero. Example: 2/3 is a rational number because 3 and 2 are both integers

Theorem: The square-root of 2 ($\sqrt{2}$) is not a rational number, i.e. it cannot be written as a fraction.

Proof: Let us assume that $\sqrt{2} = \frac{p}{q}$ where p and q are some whole numbers and the fraction $\frac{p}{q}$ cannot be simplified further. We can write:

$$\left(\sqrt{2}\right)^2 = \left(\frac{p}{q}\right)^2$$

$$2 = \frac{p^2}{q^2}$$

So that:
$$2q^2 = p^2$$

Thus, p must be an even number and could be rewritten as: $p = 2m$. Substituting:

$$2q^2 = p^2 = 4m^2$$

So that:
$$q^2 = 2m^2$$

Thus, q must be an even number. This *contradicts* our initial assertion that $\frac{p}{q}$ could not be simplified further (at least each p, q could be reduced by one factor of 2 each). Therefore, we have proven by contradiction that $\sqrt{2}$ cannot be written as a rational number.

Homework

1. Simplify:
   (a) $\left(\sqrt{17}\right)^2$

   (b) $\left(\sqrt{13}\right)^4$

   (c) $\left(\sqrt{11}\right)^3$

   (d) $\left(\sqrt{3^4 3^6}\right)$

   (e) $\left(\sqrt{2^4 3^5}\right)$

2. Can one cut a square with the side of 1m from the circle of diameter 1.4m?

3. * (optional) The side of an equilateral triangle is 1m. Find its height and area. Reminder: an equilateral triangle has all sides the same length.

4. Take a positive number $x < 100$ and using a calculator (or computer) calculate the number $\frac{x}{2} + \frac{1}{x}$. Call the result *x* and repeat the same calculation with the new *x*. Do it 10 times. Then take the result and square it. What did you get? Try to do the same thing starting with a different number. Is it surprising?

5. How many behemoths can one truck carry with a maximum load of 5 tonnes (5000 kg) if the weight of each behemoth is 1500 kg? How many crocodiles can the same truck carry if the weight of each crocodile is 175 kg?

6. Calculate:

   (a) $(2^{-1})^2$ $(3^{-2})^{-2}$ $((-2)^{-1})^2$ $\left(\left(3\frac{1}{7}\right)^0\right)^{-6}$

   (b) $\left(\frac{2(a+1)^3(a+1)^4}{3(a+1)^3}\right)^{-1}$

   (c) $2a^{-1} + 8(2a)^{-1} - 4\left(\frac{a^5}{a^4}\right)^{-1}$

7.

Simplify the following expressions:

(a)

$$\frac{1}{x+1} - \frac{1}{x-1}$$

(b)

$$\left(1 + \frac{1}{x}\right) \div (x+1)$$

(c)

$$\left(1 + \frac{1}{x}\right) \div \left(1 - \frac{1}{x}\right)$$

8. Base 16 numbers:

a) add two base 16 numbers together:

$$\begin{array}{r} ABCD \\ +\,F23E \\ \hline \end{array} \qquad \begin{array}{r} 3FBC \\ +\;A9F8 \\ \hline \end{array}$$

b) subtract two base 16 numbers :

$$\begin{array}{r} FDCB \\ -\,9ABC \\ \hline \end{array} \qquad \begin{array}{r} F35D \\ -\,9C8A \\ \hline \end{array}$$

9* Problem we discussed in class. On the table in front of Avya there are 1000 quarters, 990 tails and 10 heads. Avya is blindfolded and cannot tell the difference between a head and a tail. Help Avya to split the coins in 2 groups, so that the number of heads in each group is the same.

Additional problems that were optional for HW14, but are mandatory now:

2. Can you find a right triangle where all sides are whole numbers and the hypotenuse is 13?

3. If, in a right triangle, one leg has length 1 and the hypotenuse has length 2, what is the other leg?