Math 5B: Classwork 14
Homework #14 is due January 24.

Square-Root

The square-root of a is a number whose square is equal to a. For example: the square-root of 25 is 5 because $5^2 = 25$. Notation: square-root of a number, a, is commonly denoted as $\sqrt{a}$.
Similarly tob^n $(ab)^n = a^n b^n$,$\sqrt{ab} = \sqrt{a}\sqrt{b}$.
For example, $\sqrt{36} = \sqrt{9 \times 4} = \sqrt{9} \times \sqrt{4} = 3 \times 2 = 6$. And we also know that $\sqrt{36} = 6$.

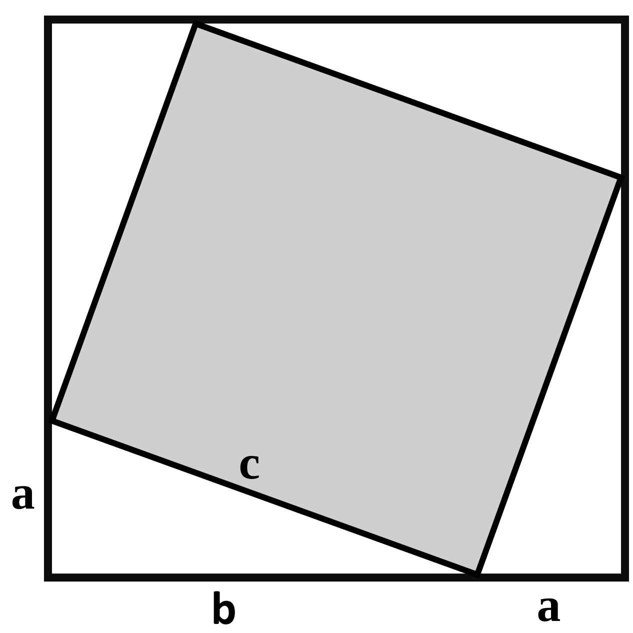
Difference of squares formula: $(x - a)(x + a) = (x^2 - a^2)$
Square of the difference formula: $(a - b)(a - b) = (a - b)^2 = a^2 - 2ab + b^2$
Square of the sum formula: $(a + b)(a + b) = (a + b)^2 = a^2 + 2ab + b^2$

Theorem (Pythagorean theorem). In a right triangle with legs a, b and hypotenuse c, one has:

$$a^2 + b^2 = c^2$$

$$c = \sqrt{a^2 + b^2}$$

A proof of this theorem is illustrated below:



In this square, the ***total area*** is:

$$(a + b) \times (a + b) = a^2 + 2ab + b^2$$

Also, the area of each small triangle is $\frac{1}{2}ab$ and the area of the shaded area is c^2 such that the total area can also be written as:

$$a^2 + 2ab + b^2 = 4 \times \frac{1}{2}ab + c^2$$
$$= 2ab + c^2$$
$$a^2 + b^2 = c^2$$

For example, in a square with side 1, the diagonal has length $\sqrt{2}$.

It is possible – but not easy – to find a right triangle where all the sides are whole numbers. The easiest such triangle is one with a,b,c = 3,4,5.

Homework

Problems 2-4 are optional since the material has not been covered in class yet. Problems 2,3,7, and 8 from the previous homework are now mandatory (listed at the end).

1. Find the following square-roots: If you cannot find the number exactly, at least say between which two whole numbers the answer is (e.g. between 5 and 6)
   (a) $\sqrt{49}$

   (b) $\sqrt{169}$

   (c) $\sqrt{225}$

   (d) $\sqrt{121}$

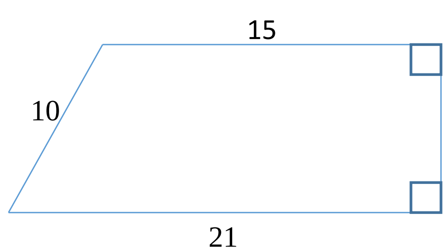
   (e) $\sqrt{64}$

   (f) $\sqrt{8}$

2. *Can you find a right triangle where all sides are whole numbers and the hypotenuse is 13?

3. * If, in a right triangle, one leg has length 1 and the hypotenuse has length 2, what is the other leg?

4. * Find the height and area of the figure: Three sides are given and the two marked angles are right angles.



5. Find the following square-roots. If you cannot find the number exactly, at least say between which two whole numbers the answer is, e.g. between 5 and 6.
   (a) $\sqrt{91+9}$
   (b) $\sqrt{42+2}$
   (c) $\sqrt{36}+\sqrt{49}$

(d) $\sqrt{49} - \sqrt{144}$

(e) $\sqrt{11^2}$

(f) $\left(\sqrt{11}\right)^2$

(g) $\left(\sqrt{64}\right)^7$

6. A watermelon is three times as expensive as a honeydew. John can buy 2 watermelons and have 7 dollars left or 4 honeydews and have 13 dollars left. How much does the honeydew cost? How much is the watermelon?

7. Yesterday, Peter came to the store and gave the cashier 11 dollars for 3 pounds of grapes; he received some change. Today, Peter came to the same store again and gave the cashier 15 dollars for 5 pounds of grapes. He again received some change. How much does each pound of grapes cost, if the change he received is the same on both days?

8. Factor the following number into primes: $99^2 - 9^2$. [Hint: you do not have to compute this number.]

9. Can you find whole numbers a; b such that $a^2 - b^2 = 17$? [Hint: use the formula we talked about in class, and think what $a - b$ and $a + b$ must be.]

10. Solve the following equations:

(g) $4\left(x - \frac{1}{6}\right) = \frac{4}{5}(x + 5) - 17$

(h) $3(x - 9) - 5(x + 11 - 20) = -1$

(i) $7 - \frac{x-6}{x+9} = 3 + \frac{-2x-15}{x+9}$

Powers of 4

n	*0*	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*
4^n	***1***	***4***	***16***	***64***						

11. Base 4 numbers:

a) add two base 4 numbers together:

$$\begin{array}{r} 321 \\ \underline{+\ 223} \end{array} \qquad \begin{array}{r} 2311 \\ \underline{+\ 3332} \end{array}$$

$$\begin{array}{r} 321 \\ \underline{-\,223} \end{array} \qquad\qquad \begin{array}{r} 3311 \\ \underline{-2222} \end{array}$$

*[*Do not add in base 10 and translate the result to base 4, try performing addition in base 4, think base 4*]*

b) Write a formula, instruction, or algorithm on how to translate base 4 number abcd to base 10 number, where a, b, c, d can be 0, 1, 2, or 3.

c) Translate the numbers and the results from a) into the base-10 system

Problems from Homework 13 (2,3,7,8) below that were deemed optional are now mandatory:

2. Do the following arithmetic operations with binary numbers. Do them without converting the numbers to decimal form:

(a) 110101**b** + 111011**b**

(b) 10101**b** × 1011**b**

(c) (10101**b** + 1101**b**) × 10110**b**

3. Base 4 numbers:

add two base 4 numbers together:

$$\begin{array}{r} 123 \\ \underline{+\,321} \end{array} \qquad\qquad \begin{array}{r} 3201 \\ \underline{+\,2310} \end{array}$$

*[Do not add in base 10 and translate the result to base 4, try performing addition in base 4, **think base 4**]*

7. Find the following square-roots. If you cannot find the number exactly, at least say between which two whole numbers the answer is, e.g. between 5 and 6.

(h) $\sqrt{16}$

(i) $\sqrt{81}$

(j) $\sqrt{10{,}000}$

(k) $\sqrt{10^8}$

(l) $\sqrt{50}$

8. Find (Hint, you do not need to compute the number under the $\sqrt{}$)

(a) $\sqrt{2^6 \times 7^2}$

(b) $\sqrt{\frac{1}{16}}$

(c) $\sqrt{\frac{4}{9}}$