Math 5B: Classwork & Homework 12 December 20, 2020
(Due on January 10)

Review

Powers: $a^n = a \times a \times a \times ... \times a$ (*n* times)

$a^0 = 1$ read: *a*-to-the-zero
$a^1 = a$ is just itself '*a*'
$(ab)^n = a^n \times b^n$
$a^n a^m = a^{n+m}$
$\frac{a^n}{a^m} = a^{n-m}$
$a^n = \frac{1}{a^{-n}}$, $a^{-n} = \frac{1}{a^n}$

1. If $a = 2^{19}3^{-91}$ and $b = 2^{-46}3^{-68}$what is the value of *ab*? of *a/b*?

Difference of squares formula: $(x - a)(x + a) = x^2 - a^2$
Square of the difference formula: $(a - b)(a - b) = (a - b)^2 = a^2 - 2ab + b^2$
Square of the sum formula: $(a + b)(a + b) = (a + b)^2 = a^2 + 2ab + b^2$

Binary Numbers: Numbers represented by using only 0s and 1s.

Powers of 2

n	*0*	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*
2^n	***1***	***2***	***4***	***8***	***16***	***32***	***64***	***128***	***256***	***512***

Example: Numbers in decimal notation can be presented like this (same as converting a number to a decimal notation): $351 = 3 \times 100 + 5 \times 10 + 1 \times 1$
Similarly, to convert a number into a binary, we need to represent it in powers of 2:

351 = **256** + 95 = **256 + 64** + 31 **= 256 + 64 + 16** + 15 **= 256 + 64 + 16 + 8** + 7 = **256 + 64 + 16 + 8 + 4 + 2 + 1**

$351 = 1\times2^8 + 0\times2^7 + 1\times2^6 + 0\times2^5 + 1\times2^4 + 1\times2^3 + 1\times2^2 + 1\times2^1 + 1\times2^0 = 101011111$

To convert number from binary to decimal we use the familiar rule where we multiply each digit by the position value in base 2. For example:

$1010 = 1\times2^3 + 0\times2^2 + 1\times2^1 + 0\times2^0 = 8 + 0 + 2 + 0 = 10$

Classwork

1. Simplify the following and show the answer in the exponent (power) form

(a) $\frac{3^7 \cdot 2^7}{2^3 \cdot 2^4} =$ (b) $\frac{6^5 \cdot 2^4}{3^5 \cdot 2^2} =$

(c) $\frac{7^9 \cdot 2^5}{7^2 \cdot 2^4} =$ (d) $\frac{11^4}{11^2 \cdot 5^2 \cdot 5^3} =$

2. Solve the equations:

(a) $\frac{9}{13} z = 3$ (b) $|x| = 2$ (c) $|x + 11| = 2$

(d) $\frac{x}{2} + 1 = \frac{4x}{7}$ (e) $x = \frac{1}{4}x + 6$ (f) $\frac{x+3}{x+1} = 4$

3. Convert the decimal numbers to binary:
   9, 12

4. Convert the following binary numbers to decimal:
   101, 1001

5. Base 4 numbers:

   a) add two base 4 numbers together:

$$\begin{array}{r} 123 \\ +\,321 \\ \hline \end{array} \qquad \begin{array}{r} 3201 \\ +\,2310 \\ \hline \end{array}$$

   *[Do not add in base 10 and translate the result to base 4, try performing addition in base 4, **think base 4**]*

Homework 12 problems

1. Solve the following equations:
   a) $3(x - 1) - 4 = 3x + 8$
   b) $\frac{1}{2}(x - 1) = -19$
   c) $|2x| = 10$

2. Simplify the following and show the answer in the exponent (power) form

(e) $7^4 \cdot 11^2 \cdot 11^{-5} \cdot 7^2 =$ (f) $\frac{3^{-5} \cdot 2^7}{3^{-3} \cdot 2^4} =$

(g) $\frac{42^2}{6^2} =$ (h) $\frac{3^5 \cdot 3^{-5}}{3^9} =$ (i) $\frac{x^2 \cdot y^2 \cdot x^{-3}}{x^2}$

3. Convert the decimal numbers to binary:
   9, 12, 24, 38, 45

4. Convert the following binary numbers to decimal:
   101, 1001, 10110, 11011, 10101

5. Factor the following number into primes: $99^2 - 9^2$. [Hint: you do not have to compute this number.]

6. Can you find whole numbers a; b such that $a^2 - b^2 = 17$? [Hint: use the formula we talked about in class, and think what $a - b$ and $a + b$ must be.]

7. For the following problem, you need to know that the speed of light is about 300,000 km/sec, and one year is about 3×10^7 seconds.
   a) In astronomy, a common unit of distance is a light year: the distance light covers in one year. How many kilometers is it?
   b) Another common unit of distance in astronomy is a parsec, which is approximately equal to 3×10^{13} km.
      - Can you compute how many parsecs are there in one light year?
      - How many parsecs is the distance between Earth and Sun (The distance is about 1.5×10^8km)?
      - How many parsecs is the distance between Earth and the Andromeda Nebula (2,000,000,000,000,000,000,000 km)?

8. If $a = 3^{19}5^{-91}$, $b = 2^{-46}3^{-68}$, $c = 2^{54}5^{-8}$, and $d = 10^{7}2^{-4}3^{-5}$ what is the value of *ab*? of *a/b*? abc? ab/c? abcd? ab/cd?

9. How many zeroes does the number $4^{15}5^{26}$ end with?

10*. This problem is optional and will be carried over to the next homework once we cover the material in class.

Base 4 numbers:

a) add two base 4 numbers together:

$$\begin{array}{r} 123 \\ +\,331 \\ \hline \end{array} \qquad \begin{array}{r} 3201 \\ +\,2313 \\ \hline \end{array}$$

*[Do not add in base 10 and translate the result to base 4, try performing addition in base 4, **think base 4**]*

b) Write a formula, instruction, or algorithm on how to translate base 4 number ***abcd*** to base 10 number, where ***a***, ***b***, ***c***, ***d*** can be 0, 1, 2, or 3.

c) Translate the numbers and the results from a) into the base-10 system