

Power Rules

General notation (n is a whole number):

$$a^n = a \times a \times a \times \dots \times a \text{ (} n \text{ times)}$$

Special cases:

$a^0 = 1$	read: a -to-the-zero
$a^1 = a$	is just itself ' a '
$a^2 = a \times a$	read: a -squared
$a^3 = a \times a \times a$	read: a -cubed

Properties:

$$(ab)^n = ab \times ab \times ab \times \dots \times ab \text{ (} n \text{ times)}$$

$$(ab)^n = (a \times a \times a \times \dots \times a) \times (b \times b \times b \times \dots \times b) \text{ (} n \text{ times)}$$

$$(ab)^n = a^n \times b^n$$

Similarly:

$$a^n a^m = (a \times a \times a \dots) \times (a \times a \times a \dots) \text{ (} n \text{ and } m \text{ times, respectively)}$$

$$a^n a^m = a \times a \times a \dots \times a \times a \text{ (} n+m \text{ times)}$$

$$a^n a^m = a^{n+m}$$

$$\frac{a^n}{a^m} = a^{n-m}$$

$$a^n = \frac{1}{a^{-n}}$$

$$a^{-n} = \frac{1}{a^n}$$

Review***PEMDAS***

Precedence rules:

P – parenthesis

E - exponents

M, D – multiplication, division

A, S – addition, subtraction

Classwork

1. Tanvi took five math tests, each worth a maximum of 100 points. Tanvi's score on each test was an integer between 0 and 100, inclusive. Tanvi received the same score on the first four tests, and she received a higher score on the last test. Her average score on the five tests was 82. How many values are possible for Tanvi's score on the last test?

Range (0;100) (round brackets) – means 0 and 100 are not included

Range [0;100] (square brackets) – values of 0 and 100 are included

1. Given:

- 5 tests, average 82; integer scores only
- each test between [0;100]
- same score on first 4 tests, higher score on the last test

2. Question: How many values are possible on the last test?

3. Solution plan (solution algorithm):

- 1) Find the total points (410)
- 2) Determine the range of possible scores for the first 4 tests – [78;81]
 - a. Determine the lowest possible score: 78
 - b. Determine the highest possible score; 81
- 3) Determine possible scores for the last test:

4. Implement the solution:

- a. $5 * 82 = 410$ – total scores
- b. Determine range for the first 4 tests: [78;81]
- c. Determine the range for the last test:

$$410 - 78 * 4 = 98$$

$$410 - 81 * 4 = 86$$

Homework (added problems in bold and #11)

1. Solve the following equations:

(a) $5 - x = -4 - 2x$

(b) $7 - 2(1 - x) = -5$

(c) $\frac{x-13}{x+3} = 5$

(d) $\frac{x-6}{x+7} + 9 = 3$

2. If you take half my age and add 7, you get my age 13 years ago. How old am I?

3. Simplify:

(a) $\frac{(x^2y^2)x^3}{x^2y^5}$

(b) $(3y^3 \cdot y^5)^2$

(c) $(4c^{-5} \cdot c^3)^7$

(d) $(2zab^4 \cdot 4a^{-3} \cdot z)^5$

4. Let $a = 2 \cdot 10^8$, $b = 10^5$. Compute $a^2 \cdot b$, $\frac{a}{b}$, $a^2 \div b^3$ (Hint: use $(a \cdot b)^n = a^n b^n$ and $(a^n)^m = a^{mn}$)

5. How many cubic centimeters are there in one cubic kilometer? (1km = 1000m, 1m=100cm)

6. It is known that $2^{10} = 1024$, which is very close to 10^3 . Use this to estimate the value of 2^{20} , 2^{32}

7. Evaluate:

(a) $(x - 5)(2x + 1) =$

(b) $(x + 7)(x^2 - 2x) =$

8. Solve:
- (a) $2^{-2} \cdot (2^2 + 4^2) =$
- (b) $6^3 \cdot (2^{-3} + 3^{-3}) =$
9. *One can measure temperature using either the Fahrenheit scale (common in the US and Britain) or the Celsius scale (in most other countries). The relation between the two is given by
- $$C = \frac{5}{9}(F - 32) \quad [C \text{ in the temperature in Celsius, } F \text{ – in Fahrenheit}]$$
- (a) Is there a temperature which gives the same value on both scales ($F = C$)?
- (b) Is there a temperature which in Fahrenheit scale is twice as large as in Celsius ($F = 2C$)?
10. There are three buckets: 10 liters, 4 liters, and 3 liters. The 10-liter bucket is full of water. There is no other water available. Divide the water so that there is exactly 5 liters in the 10-liter bucket, 1 liter in the 3-liter bucket, and 4 liters in the 4-liter bucket. You may only pour back and forth between the three given buckets. Describe how to do that using a table below. First and last columns are done for you.

10-l bucket						5
4-l bucket						4
3-l bucket						1

11. Brian took 10 math tests, each worth a maximum of 100 points. Brian's score on each test was an integer between 0 and 100, inclusive. Brian received the same score on the first 9 tests and a lower score on the last test. None of his test scores were below 70. His average score on the ten tests was 90. How many values are possible for Brian's score on the first 9 tests and on the last test?