Powers of 2, Power rules

Example 1: if a certain population of bacteria doubles every day, and right now we have 1 gram of them, how much will we have in 2 days? In a week? In a month?

The answer: after 1 day we would have 2 grams.
after 2 days we would have $2 \times 2 = 4$ grams.
after 3 days we would have $2 \times 2 \times 2 = 8$ grams.
after 4 days we would have $2 \times 2 \times 2 \times 2 = 16$ grams.
after n days we would have $2 \times 2 \times 2 \ldots \times 2$ $(n\ 2's)$ grams

There is a special notation for this: $2^n = 2 \times 2 \times 2 \ldots \times 2$ $(n\ 2's)$ (pronounced *2-to-the-n*)

This grows very quickly: for *n=10* (in ten days) we will have $2^{10} = 1024$ grams; in another 10 days, the amount will again multiply by 1024, so we will have $1024 \times 1024 \approx 1000000$grams, or one ton of bacteria; in 30 days (one month) we will have 1000 tons!

Example 2: In some problems, instead of multiplying by 2 every time, we divide by 2 every time.
Problem: a friend thinks of a number between 1 and 100, you try to deduce the number by asking questions that can only be answered by 'yes' or 'no'.
Solution: the best strategy is to ask a question which cuts the number of possibilities in half. So the first question should be: 'Is the number larger than 50?' If the answer is 'yes' we know that the number is between 51 and 100; if it is 'no' then is it between 1 and 50. Either way there are only 50 possibilities left. The next question should again cut the number in half (e.g. is the number larger than 75?' After *n* guesses, the number of remaining possible numbers is $100/2^n$, so the number of needed questions is: 7 (i.e. smallest number such that: $2^n > 100$)

General notation (*n* is a whole number):

$$a^n = a \times a \times a \times \ldots \times a \text{ (} n \text{ times)}$$

Special cases:

$a^0 = 1$	read: *a*-to-the-zero
$a^1 = a$	is just itself '*a*'
$a^2 = a \times a$	read: *a*-squared
$a^3 = a \times a \times a$	read: *a*-cubed

Properties:

$$(ab)^n = ab \times ab \times ab \times \ldots \times ab \text{ (} n \text{ times)}$$
$$(ab)^n = (a \times a \times a \times \ldots \times a) \times (b \times b \times b \times \ldots \times b) \text{ (} n \text{ times)}$$
$$(ab)^n = a^n \times b^n$$

Similarly:

$a^n a^m = (a \times a \times a \dots) \times (a \times a \times a \dots)$ (*n* and *m* times, respectively)
$a^n a^m = a \times a \times a \dots \times a \times a$ (*n*+*m* times)
$a^n a^m = a^{n+m}$

Homework

1. Solve the following equations:

   (a) $5(x-1) - 4 = 3x + 1$
   (b) $\frac{2}{3}(x-2) = -18$
   (c) $|2x+1| = 7$
   (d) $-|3x - 7 + 8x| = -15$
   (e) $\frac{x-8}{11} = -35$
   (f) $\frac{x+16}{x} = -7$
   (g) $\frac{x}{x-7} = 5$
   (h) $\frac{x-6}{x-9} = 8$
   (i) $\frac{x-15}{11-x} = -12$

2. When Dennis was 27, his son was three years old. Now his son's age is one-third of Dennis' age. How old is each of the now? <u>(Solve using equation please – guess & check no longer accepted)</u>

3. Find the sum of $1 + 2 + 4 + \cdots + 2^n$ (the answer will depend on *n*). [Hint: first try computing it for several small values of n, see if a pattern emerges, then find a general rule.]

4. If we put one grain of wheat on the first square of the chessboard, two on the second, then four, eight, … approximately how many grains of wheat will there be? (You can use $2^{10} = 1024 \approx 1000$.)

5. Lotus flowers are growing in a lake. Every day each lotus plant divides into two plants, so the area its leaves cover is doubled. In 30 days the whole lake is covered with lotus leaves. When was exactly half of the lake covered by leaves?

6. There are 15 samples of water from various wells. It is known that exactly one of them contains a dangerous chemical. A lab can test for the chemical, but the analysis is time consuming and expensive. Can you find the sample containing the chemical using fewer than 15 tests? How many tests are needed? [Hint: take a droplet of water from each sample and test the new combined sample … use the strategy in 'example 2' above.]

7. Write as powers with base 3:
   (a) 243×3^3
   (b) $3^7 \times 3^{11} \times (-81)$
   (c) $-3^5(27 - 3^4)$

8. Find the prime factorization of 500 and 1215. Express as a multiplication of powers. Find the greatest common factor (GCF).

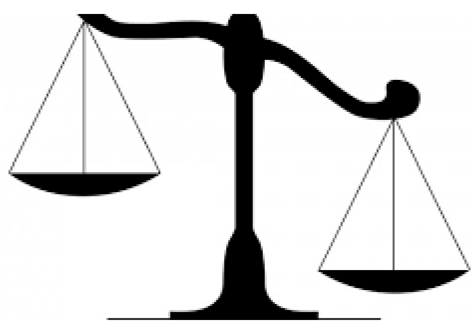
9. Cut each square on a picture below (trace with colored pencils instead of cutting) into 4 equal parts so that each part gets one "X".

	X	X	
	X	X	

X			
	X		
		X	
			X

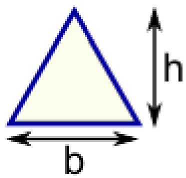
		X	
X	X	X	

10. Imagine we have N coins one of which is counterfeit and therefore its weight is different than that of the other ones. We also have a weighing balance scale that can show us whether one group of coins is heavier or lighter than the other. If we can use the balance scale only twice, can we always determine whether the counterfeit coin is lighter or heavier than the good ones? Solve for **N = 4,5, and 6.**

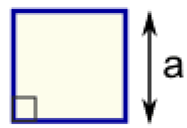


OPTIONAL

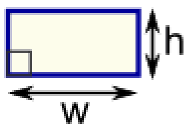
***Area** is the size of a surface!*
http://www.mathsisfun.com/area.html



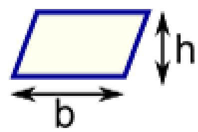
Triangle
Area = ½ × b × h
b = base
h = vertical height



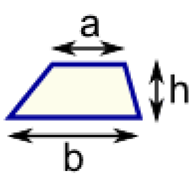
Square
Area = a^2
a = length of side



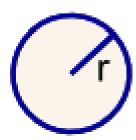
Rectangle
Area = w × h
w = width
h = height



Parallelogram
Area = b × h
b = base
h = vertical height



Trapezoid (US)
Trapezium (UK)
Area = ½(a+b) × h
h = vertical height



Circle
Area = $\pi \times r^2$
Circumference = $2 \times \pi \times r$
r = radius

11. Compute the area of the figures below. The picture is not to scale, so do not try measuring the lengths – use the numbers given.