Math 4d. Classwork 10.

Algebra.

About variables.

When we need wright the mathematical expression, but we don't know exact numbers to be used, we use variables. It can be any symbol, but it's very convenient to use letters. For example, if the number of the books on the first shelf is n and the number of the book on the second shelf is m, the total number of books on both shelves is $n + m$. We can perform all the usual arithmetic operation with the variables, but the exact answer can be reached only when the values are passed to the variables.

From the homework:

Write the expression for the following problems:

a. 3 packages of cookies cost a dollars. How many dollars do 5 of the same packages cost?

If 3 packages of cookies cost a dollars, one pack is cost

$$1pack = \frac{a}{3} = a:3$$

Five such packs will be

$$5 \cdot a:3 = \frac{5a}{3} = \frac{5}{3}a$$

b. 5 bottles of juice cost b dollars. How many bottles can one buy with c dollars?

Again, if 5 bottles cost b dollars, one bottle will cost

$$\frac{b}{5} \text{ dollars}$$

If I have only c dollars, I can buy the number of bottles equal to my total money divided by the price of one bottle:

$$c:\frac{b}{5} = c \cdot \frac{5}{b} = \frac{5c}{b}$$

If I have only \$30 and 5 bottles cost 10 dollars I can buy :

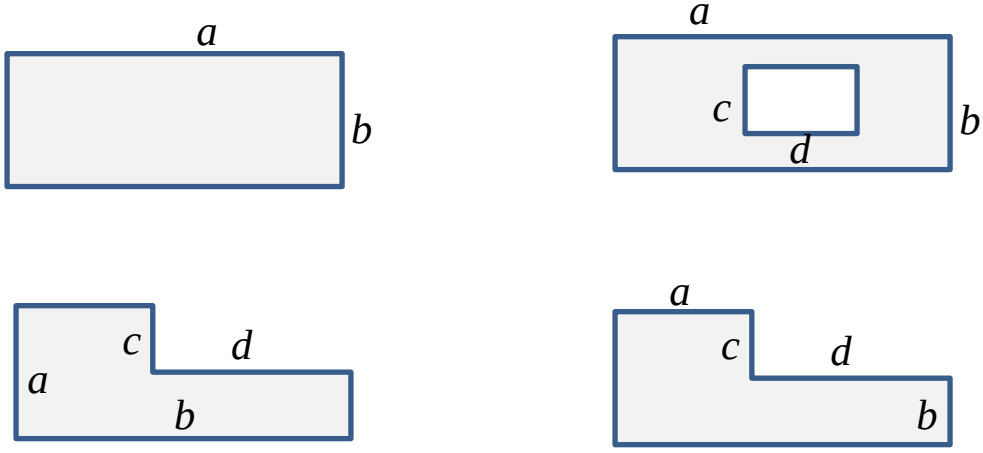
$$30:\frac{10}{5} = 30 \cdot \frac{5}{10} = 30 \cdot \frac{1}{2} = 15 \text{ bottles}$$

1. Apple costs x dollars and pear costs y dollars. Explain the expressions below:

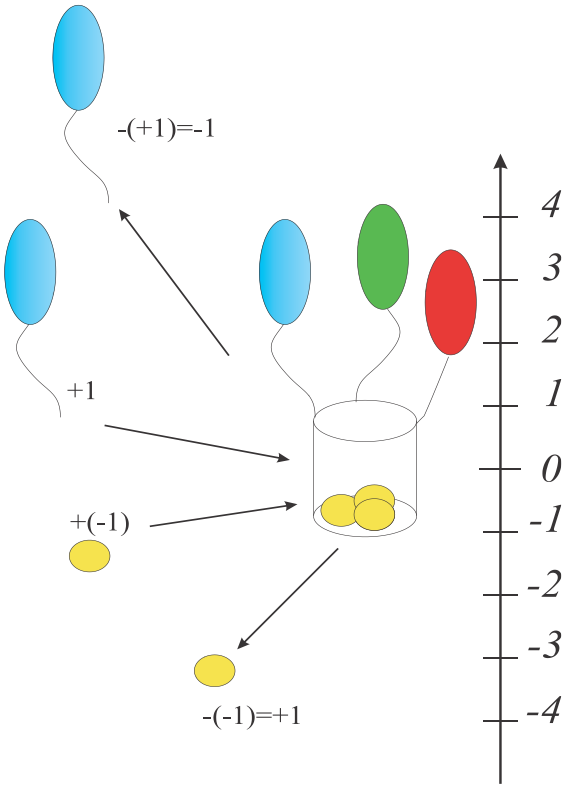
   $x + y, \quad x - y, \quad 3x, \quad 8y, \quad 3x + 8y, \quad y:x, \quad 120:y$

2. Write a single numerical expression to solve the problem:
   There are 25 students in a class. After school, 7 students went home, and the rest made 3 equal teams to play basketball. How many students are in each team?

3. Alex is m years old. Robert is n years older than Alex. How many times Alex will be younger than Robert in 3 years? Solve the problem for $m = 2, n = 10$.
4. Julia had 20 cards. She gave a cards to her sister. How many cards she has now? Can a be any number?
5. Write the expressions for the shaded areas below:



Positive and negative numbers.

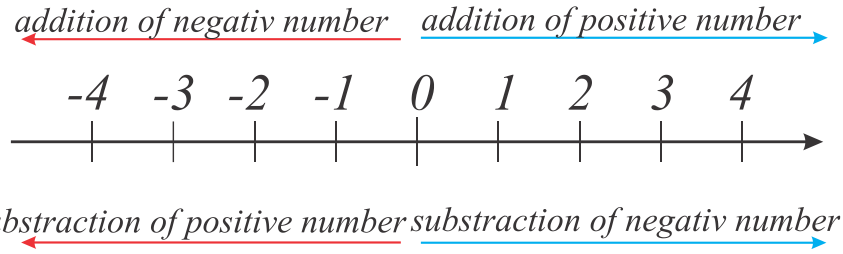


Negative numbers represent opposites. If positive represents movement to the right, negative represents movement to the left. If positive represents above sea level, then negative represents below level. If positive represents a deposit, negative represents a withdrawal. They are often used to represent the magnitude of a loss or deficiency. Negative numbers appeared for the first time in history in the Nine Chapters on the Mathematical Art, which in its present form dates from the period of the Chinese Han Dynasty (202 BC – AD 220), but may well contain much older material. Liu Hui (c. 3rd century) established rules for adding and subtracting negative numbers. By the 7th century, Indian mathematicians such as Brahmagupta were describing the use of negative numbers. Islamic mathematicians further developed the rules of subtracting and multiplying negative numbers and solved problems with negative coefficients. Western mathematicians accepted the idea of negative numbers by the 17th century. Prior to the concept of negative numbers, mathematicians such as

Diophantus considered negative solutions to problems "false" and equations requiring negative solutions were described as absurd

Last year when we discuss the negative numbers we used very clear analogy of a basket with balloons and sand bags.

At the beginning basket has N balloons and N sand bags, placed at 0 position and doesn't move. Balloons represent positive units, sand bags represent negative units. If we add one balloon the basket will move one unit up. If we add one sand bag basket will move one unit down. If we remove one balloon, basket will go one unit down, which is equivalent of adding one sand bag. So $-(+1) = +(-1)$. If we remove one sand bag, basket will go one unit up, which is equivalent of adding one balloon. So $-(-1) = +(+1)$. Let's move to number line:



Two numbers that have the same magnitude but are opposite in signs are called opposite numbers.

Exercises.

Fill up the table:

a	7	-4			5		0	
$-a$			0	-1		8		-3

1. Compare:

$-4 \quad 4$ $\qquad$ $6 \quad -4$ $\qquad$ $\frac{2}{3} \quad -\frac{3}{2}$

$-4 \quad -2$ $\qquad$ $-4 \quad 0$ $\qquad$ $-\frac{2}{3} \quad -1$

$-4 \quad -6$ $\qquad$ $-1 \quad -\frac{1}{2}$ $\qquad$ $-2 \quad \frac{1}{2}$

2. Compute:

$3 + (-2) =$ $3 + (+2) =$ $-3 - (-2) =$

$-3 + (-2) =$ $-3 + (+2) =$

$3 - (+2) =$

$-3 - (+2) =$ $-3 + (+3) =$

$3 - (-2) =$

3. Compare without calculation.

$100 - (35 - 20)$ $100 - (35 + 20)$ $100 + (35 - 20)$ $100 + (35 + 20)$

$100 - (-35 - 20)$ $100 - (-35 + 20)$ $100 + (-35 - 20)$ $100 + (-35 + 20)$

4. Rewrite without parenthesis:

$20 + (2 - 3) =$ $20 - (-2 + 3) =$

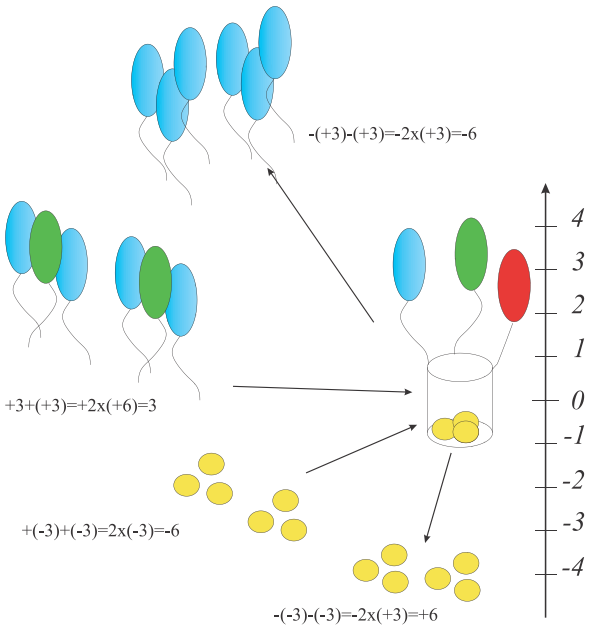
$20 - (2 - 3) =$ $20 - \big(-2 + (-3)\big) =$

Multiplication and division of negative numbers.

If we multiply 2 positive numbers we will get third positive number. What will happened if we multiply one negative and one positive number. Let's again review our analogy. In this case we will add or remove our balloons and sand bags by groups of three. Addition of two groups of 3 sand bags will drive the basket 6 units down, because we add 6 bags. So $2 \times (-3) = -6$. (We know that $-1 + (-1) + (-1) = -3$)

Removing of 2 groups of 3 sand bags will drive our basket 6 units up, which is corresponding of adding 6 balloons, so $-2 \times (-3) = 6$

Addition of 6 balloons (2 groups of three) of cause will help us to move up for 6 units. If we



remove 2 groups of 3 balloons we will descend 6 units. $-2 \times (+3) = -6$.

5. Solve the following equations:

$$x + 4 = -1 \qquad 5 - x = -3 \qquad x - (-4) = 0$$

6. Positive or negative number will be the product of
a) Two negative and one positive numbers.
b) One negative and two positive numbers
c) Three negative numbers.

7. A swimming pool can be filed by one pipe in 10 hours or by another pipe in 15 hours. How long it will take to fill up the pool with both pipes opened?

8. A swimming pool can be filed with one pipe in 10 hours. Full pool can be drain out with another pipe in 20 hours. How long it will take to fill up the pool with opened drain pipe?