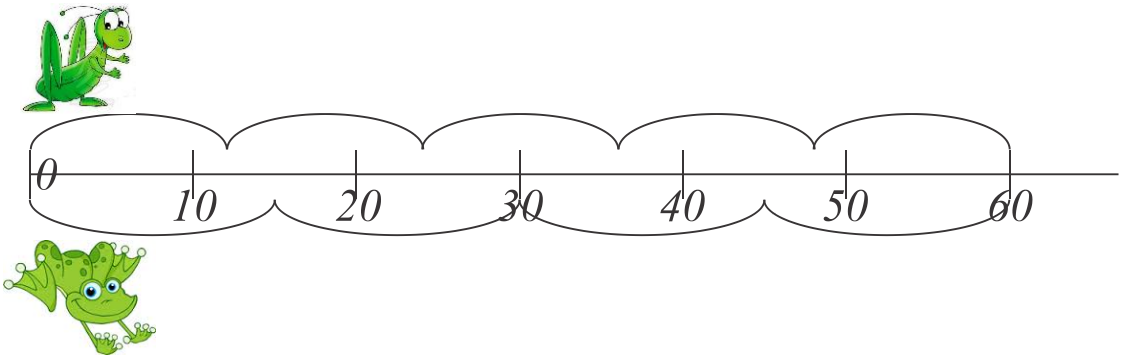
Math 4 d. Class work 6.

Algebra.

A grasshopper jumps a distance of 12 centimeters each jump. A little frog jumps a distance of 15 centimeters each jump. They start hopping at the same time from the same point 0 and jump along the big ruler. What is the closest point on the ruler at which they can meet?

There are places on the ruler that both of them can reach after some number of jumps. One of such places is, of course, 12×15 cm. A grasshopper can make 15 jumps while a little frog can make only 12 jumps. Will 12x15 be the only place where they can meet or there are some other places? If this is the case, we have to find a number that is divisible by both 12 and 15. Take into account that 12×15, as well as any product of 12×15 is divisible by both 12 and 15.

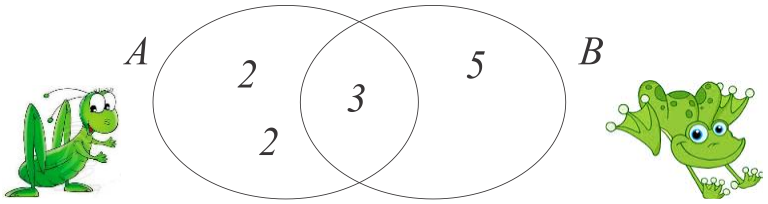


Is there are any number which is less then 12×15 and still divisible by 12 and 15?
Prime factorization of 12 and 15:

12	2	15	3
6	2	5	5
3	3	1	
1			

$$12 \times 15 = (2 \times 2 \times 3) \times (3 \times 5)$$

The number which we are looking for has to be a product of prime factors of either 12 or 15, so it should be a union of two sets – set of prime factors of 12 and 15.



$$2 \times 2 \times 3 \times 5 = 60$$

60 is the smallest number, which is divisible by 12 and 15, LCM.

The Johnson's family want to by same amount of gammy worms and mini chocolates (168 mini per box and 180 g.w per box). How many boxes they need to buy?

$7 \cdot 2 \cdot 2 \cdot 2 \cdot 3 = 168;$

$2 \cdot 2 \cdot 3 \cdot 3 \cdot 5 = 180$

$7 \cdot 2 \cdot 2 \cdot 2 \cdot 3 \cdot 3 \cdot 5 = 2520$

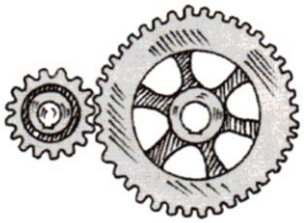
Eratosthenes proposed a simple algorithm for finding prime numbers. This algorithm is known in

mathematics as the Sieve of Eratosthenes.

In mathematics, the sieve of Eratosthenes, one of a number of prime number sieves, is a simple, ancient algorithm for finding all prime numbers up to any given limit. It does so by iteratively marking as composite, *i.e.*, not prime, the multiples of each prime, starting with the multiples of 2.

~~1~~	2	3	~~4~~	5	~~6~~	7	~~8~~	~~9~~	~~10~~
11	~~12~~	13	~~14~~	~~15~~	~~16~~	17	~~18~~	19	~~20~~
~~21~~	~~22~~	23	~~24~~	~~25~~	~~26~~	~~27~~	~~28~~	29	~~30~~
31	~~32~~	~~33~~	~~34~~	~~35~~	~~36~~	37	~~38~~	~~39~~	~~40~~
41	~~42~~	43	~~44~~	~~45~~	~~46~~	47	~~48~~	~~49~~	~~50~~
~~51~~	~~52~~	53	~~54~~	~~55~~	~~56~~	~~57~~	~~58~~	59	~~60~~
61	~~62~~	~~63~~	~~64~~	~~65~~	~~66~~	67	~~68~~	~~69~~	~~70~~
71	~~72~~	73	~~74~~	~~75~~	~~76~~	~~77~~	~~78~~	79	~~80~~
~~81~~	~~82~~	83	~~84~~	~~85~~	~~86~~	~~87~~	~~88~~	89	~~90~~
~~91~~	~~92~~	~~93~~	~~94~~	~~95~~	~~96~~	97	~~98~~	~~99~~	~~100~~

Exercises.



1. Two gears are in in clutch. One gear has 18 cogs, and another has 63. How many turns will each gear make before they both return to their original position?

2. Two buses leave from the same bus station following two different routes. For the first one it takes 48 minutes to complete the roundtrip route. For the second one it takes 1 hour and 12 minutes to complete the round trip route. How much time will it take for the buses to meet at the bus station for the first time after the have departed for their routes at the same time?

3. Find all prime factors of the following numbers:
   66, 28, 128, 555, 1233

4. Find GCD (GCF) of
   a. 420 and 450,
   b. 810, 945 and 1125

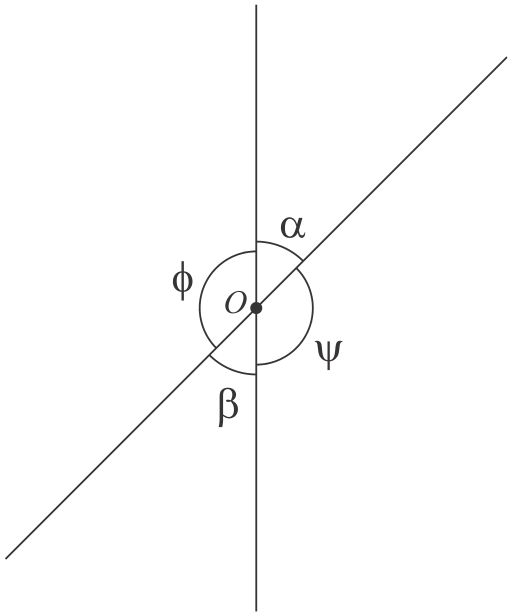
5. Find LCM of
   a. 8 and 12
   b. 15, 18, and 21

6. Is number a is divisible by number b? if yes, find the the quotient.

1) $a = 2 \cdot 2 \cdot 3 \cdot 7 \cdot 7,\ b = 2 \cdot 2 \cdot 11;$
2) $a = 2 \cdot 3 \cdot 5 \cdot 13,\ b = 5 \cdot 13;$
3) $a = 3 \cdot 5 \cdot 5 \cdot 11 \cdot 17,\ b = 3 \cdot 5 \cdot 17;$
4) $a = 2 \cdot 2 \cdot 3 \cdot 3 \cdot 5 \cdot 19 \cdot 23,\ b = 2 \cdot 2 \cdot 3 \cdot 5;$
5) $a = 2 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 5 \cdot 11 \cdot 13,\ b = 405;$
6) $a = 2 \cdot 3 \cdot 7 \cdot 11 \cdot 13 \cdot 29,\ b = 2002.$

Geometry.

When two straight lines intersect at a point, four angles are formed. A pair of angles opposite each other formed by two intersecting straight lines that form an "X"-like shape, are called vertical angles, or opposite angles, or vertically opposite angles.



α and β and ϕ and ψ are 2 pairs of vertical angles.

Vertical angles theorem:

Vertical angles are equal.

In mathematics, a **theorem** is a statement that has been proven on the basis of previously established statements.
According to a historical legend, when Thales visited Egypt, he observed that whenever the Egyptians drew two intersecting lines, they would measure the vertical angles to make sure that they were equal. Thales concluded that one could prove that vertical angles are always equal and there is no need to measure them every time.

Proof:

$\angle\phi + \angle\alpha = 180°$ because they are supplementary by construction.
$\angle\phi + \angle\beta = 180°$ because they are supplementary also by construction.

1. 4 angles are formed at the intersection of 2 lines. One of them is 40°. What is the measure of 3 others?
2. 3 lines intersect at 1 point and form 6 angles. One is 55°, another is 72°. Can you find all other angles?