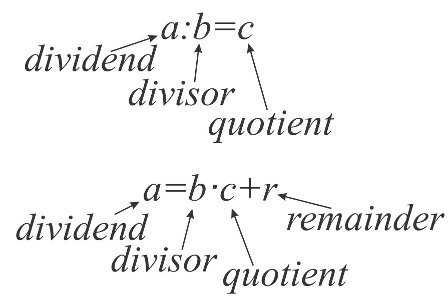
Math 4 d. Class work 4.

1. **Divisibility.**

We say that a natural number is divisible by another natural number if the result of this operation is a natural number. If this is not the case then we can divide a number with a remainder.



If a and n are natural numbers, the result of a division operation of $a \div n$ will be a quotient c, such that

$$a = b \times c + r$$

Where r is a remainder of a division $a \div b$. If r is 0, then we can tell that a is divisible by b.

- If we want to divide m by 15, what numbers we can get as a remainder?

If the remainder is 0, then quotient and divisor are both factors of dividend, $a = b \cdot c$, and to divide number ***a*** by another number, ***b***, means to find such number ***c***, that $c \cdot b$ will give us a.

So, because the product of 0 and any number is 0, then there is no such arithmetic operation as division by 0.

Divisibility Rules	
A number is divisible by	
2	If last digit is 0, 2, 4, 6, or 8
3	If the sum of the digits is divisible by 3
4	If the last two digits is divisible by 4
5	If the last digit is 0 or 5
6	If the number is divisible by 2 and 3
7	cross off last digit, double it and subtract. Repeat if you want. If new number is divisible by 7, the original number is divisible by 7
8	If last 3 digits is divisible by 8
9	If the sum of the digits is divisible by 9
10	If the last digit is 0
11	Subtract the last digit from the number formed by the remaining digits. If new number is divisible by 11, the original number is divisible by 11
12	If the number is divisible by 3 and 4

Factorization.

In mathematics factorization is a decomposition of one number into a product of two or more numbers, or representation of an expression as a product of 2 or more expressions, which called 'factors'. For example, we can represent the expression $a \cdot b + a \cdot c$ as a product of a and expression $(b + c)$. Can you explain why?

$$a \cdot b + a \cdot c = a \cdot (b + c)$$

Or in a numerical expression:

$$7 \cdot 5 + 7 \cdot 3 = 7 \cdot (5 + 3)$$

Or a number can be representing as product of two or more other numbers, for example:

$$40 = 4 \cdot 10 = 4 \cdot 2 \cdot 5, \quad 36 = 6 \cdot 6 = 3 \cdot 2 \cdot 6$$

Does any natural number can be represented as a product of 2 or more numbers besides 1 and itself? Natural numbers greater than 1 that has no positive divisors other than 1 and itself are called prime numbers.

Even numbers are the numbers divisible by 2 (they have 2 as a divisor), so they can be factorized as 2 times something else. Can an even number be a prime number? Is there any even prime number?

Prime factorization or integer **factorization** of a number is the determination of the set of **prime** numbers which multiply together to give the original integer. It is also known as **prime** decomposition.

Prime factorization process:

Prime factors of 168 are $2, 2, 2, 3, 7$ and prime factors of 180 are $2, 2, 3, 3, 5$,

168	2
84	2
42	2
21	3
7	7
1	

180	2
90	2
45	3
15	3
5	5
1	

$$2 \times 2 \times 2 \times 3 \times 7 = 168; \quad 2 \times 2 \times 3 \times 3 \times 5 = 180$$

Eratosthenes proposed a simple algorithm for finding prime numbers. This algorithm is known in mathematics as the Sieve of Eratosthenes.

In mathematics, the sieve of Eratosthenes, one of a number of prime number sieves, is a simple, ancient algorithm for finding all prime numbers up to any given limit. It does so by iteratively marking as composite, *i.e.*, not prime, the multiples of each prime, starting with the multiples of 2.

~~1~~	2	3	~~4~~	5	~~6~~	7	~~8~~	~~9~~	~~10~~
11	~~12~~	13	~~14~~	~~15~~	~~16~~	17	~~18~~	19	~~20~~
~~21~~	~~22~~	23	~~24~~	~~25~~	~~26~~	~~27~~	~~28~~	29	~~30~~
31	~~32~~	~~33~~	~~34~~	~~35~~	~~36~~	37	~~38~~	~~39~~	~~40~~
41	~~42~~	43	~~44~~	~~45~~	~~46~~	47	~~48~~	~~49~~	~~50~~
~~51~~	~~52~~	53	~~54~~	~~55~~	~~56~~	~~57~~	~~58~~	59	~~60~~
61	~~62~~	~~63~~	~~64~~	~~65~~	~~66~~	67	~~68~~	~~69~~	~~70~~
71	~~72~~	73	~~74~~	~~75~~	~~76~~	~~77~~	~~78~~	79	~~80~~
~~81~~	~~82~~	83	~~84~~	~~85~~	~~86~~	~~87~~	~~88~~	89	~~90~~
~~91~~	~~92~~	~~93~~	~~94~~	~~95~~	~~96~~	97	~~98~~	~~99~~	~~100~~

1. Proof that the sum of two any even natural numbers is an even number.
2. The remainder of 1932 ÷ 17 is 11, the remainder of 261 ÷ 17 is 6. Is 2193 = 1932 + 261 divisible by 17? Can you tell without calculating? Explain.
3. Find all natural numbers such that upon division by 7 the quotient and remainder will be equal.
4. Even or odd number will be the sum and the product of
   a. 2 odd numbers
   b. 2 even numbers
   c. 1 even and 1 odd number
   d. 1 odd and 1 even number

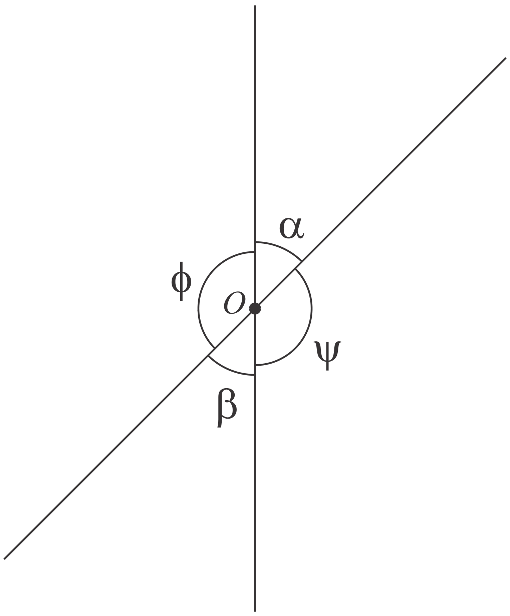
   Can you explain why? (a few examples do not prove the statement).
5. Even or odd number will be the sum

   $1 + 2 + 3 + \ldots + 10$

   $1 + 2 + 3 + \ldots + 100$

   $1 + 2 + 3 + \ldots + 1000$

When two straight lines intersect at a point, four angles are formed. A pair of angles opposite each other formed by two intersecting straight lines that form an "X"-like shape, are called vertical angles, or opposite angles, or vertically opposite angles.



α and β and ϕ and ψ are 2 pairs of vertical angles.

Vertical angles theorem*:*

Vertical angles are equal.

In mathematics, a **theorem** is a statement that has been proven on the basis of previously established statements.

According to a historical legend, when Thales visited Egypt, he observed that whenever the Egyptians drew two intersecting lines, they would measure the vertical angles to make sure that they were equal. Thales concluded that one could prove that vertical angles are always equal and there is no need to measure them every time.

Proof:

$\angle\phi + \angle\alpha = 180°$ because they are supplementary by construction.

$\angle\phi + \angle\beta = 180°$ because they are supplementary also by construction.

1. 4 angles are formed at the intersection of 2 lines. One of them is 30°. What is the measure of 3 others?
2. 3 lines intersect at 1 point and form 6 angles. One is 44°, another is 38°. Can you find all other angles?