Math 4 d. Class work 3.

1. **Properties of the arithmetic operations.**

Addition

Multiplication

Commutative property

$a+b=b+a$

$a \times b=b \times a$

Associative property

$a+(b+c)=(a+b)+c$

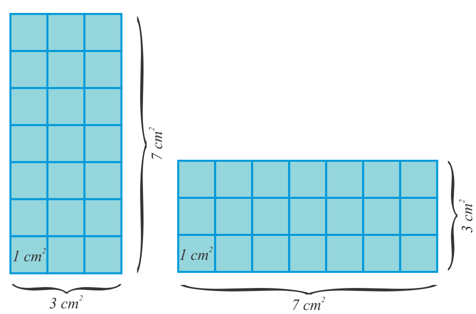
$a \times (b \times c)=(a \times b) \times c$

Distributive property

$a \times (b+c)=a \times b + a \times c$

$a \times (b-c)=a \times b - a \times c$

Commutative and associative properties of addition are easy to understand. Multiplication is just a shorter way to write the addition of equal groups, so commutative and associative properties of multiplication can be visualized and understood with the help of the rectangle area. (See the piture). Areas of identical rectangles are equal,



$$S = 3cm^2 \cdot 7 = 7cm^2 \cdot 3 = 3cm \cdot 7cm = 21cm^2$$

The distributive property can be explained with the definition of multiplication as well;

$2 \cdot (3+7) = (3+7) + (3+7) = 3+3+7+7 = 2 \cdot 3 + 2 \cdot 7$ and it is true for any numbers.

We can do it he other way around:

$$2 \cdot 3 + 2 \cdot 7 = 3+3+7+7 = 3+7+3+7 = (3+7)+(3+7) = 2 \cdot (3+7)$$

The distributive property can be illustrated by the following problems:

Farmer put green and red grapes into boxes. Each box contains 5lb of grapes. How many pounds of green and red grapes altogether did farmer put into boxes if he had 10 boxes of green and 8 boxes of red grapes? Is there any difference between 2 following expressions?

$$5 \cdot (10+8) \text{ or } 5 \cdot 10 + 5 \cdot 8$$

What is represented by the first expression? By the second?

Another example:

For the party John bought 7 identical boxes of chocolates, 20 candies in each box.

Guests ate 12 candies from each box. How many chocolates are left after the party?

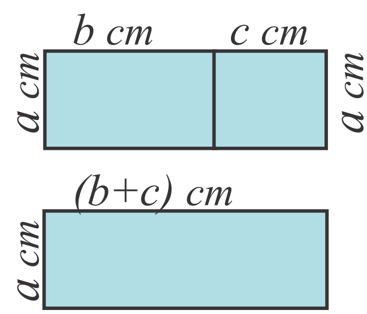
Again, two numerical expression can be written to describe the problem:

$$7 \cdot (20-12) \text{ and } 7 \cdot 20 - 7 \cdot 12.$$

For both examples we can write the equality:

$$7 \cdot (20-12) = 7 \cdot 20 - 7 \cdot 12$$
$$5 \cdot (10+8) = 5 \cdot 10 + 5 \cdot 8$$

These equalities are numerical representation of the distributive property, which can be written in the general form as $a \cdot (b+c) = a \cdot b + a \cdot c$. (and of cause $a \cdot b + a \cdot c = a \cdot (b+c)$ is also true, this way of writing the distributive property is called the factoring the common factor out (of the parenthesis). The other way to see the distributive property is as an combined area of two rectangles with one side of the same length and the area of one rectangle. Combined area of two rectangles S_2 equals to

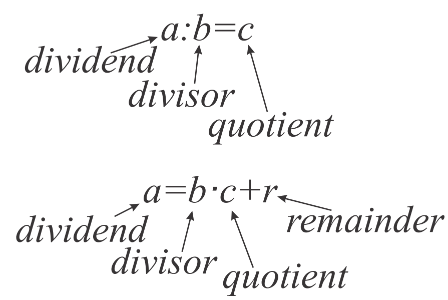


$a \cdot b + a \cdot c$, and the area of one big rectangle is $S_1 = a(b + c)$:

$$S_1 = a(b + c) = a \cdot b + a \cdot c = S_2$$

2. Divisibility.

We say that a natural number is divisible by another natural number if the result of this operation is a natural number. If this is not the case then we can divide a number with a remainder.



If a and n are natural numbers, the result of a division operation of $a \div n$ will be a quotient c, such that

$$a = b \times c + r$$

Where r is a remainder of a division $a \div b$. If r is 0, then we can tell that a is divisible by b.

- If we want to divide m by 15, what numbers we can get as a remainder?

If the remainder is 0, then quotient and divisor are both factors of dividend, $a = b \cdot c$, and to divide number $\boldsymbol{a}$ by another number, $\boldsymbol{b}$, means to find such number $\boldsymbol{c}$, that $c \cdot b$ will give us a.

So, because the product of 0 and any number is 0, then there is no such arithmetic operation as division by 0.

Divisibility Rules	
A number is divisible by	
2	If last digit is 0, 2, 4, 6, or 8
3	If the sum of the digits is divisible by 3
4	If the last two digits is divisible by 4
5	If the last digit is 0 or 5
6	If the number is divisible by 2 and 3
7	cross off last digit, double it and subtract. Repeat if you want. If new number is divisible by 7, the original number is divisible by 7
8	If last 3 digits is divisible by 8
9	If the sum of the digits is divisible by 9
10	If the last digit is 0
11	Subtract the last digit from the number formed by the remaining digits. If new number is divisible by 11, the original number is divisible by 11
12	If the number is divisible by 3 and 4

Factorization.

In mathematics factorization is a decomposition of one number into a product of two or more numbers, or representation of an expression as a product of 2 or more expressions, which called 'factors'. For example, we can represent the expression $a \cdot b + a \cdot c$ as a product of a and expression $(b + c)$. Can you explain why?

$$a \cdot b + a \cdot c = a \cdot (b + c)$$

Or in a numerical expression:

$$7 \cdot 5 + 7 \cdot 3 = 7 \cdot (5 + 3)$$

Or a number can be representing as product of two or more other numbers, for example:

$$40 = 4 \cdot 10 = 4 \cdot 2 \cdot 5, \quad 36 = 6 \cdot 6 = 3 \cdot 2 \cdot 6$$

Does any natural number can be represented as a product of 2 or more numbers besides 1 and itself? Natural numbers greater than 1 that has no positive divisors other than 1 and itself are called prime numbers.

Even numbers are the numbers divisible by 2 (they have 2 as a divisor), so they can be factorized as 2 times something else. Can an even number be a prime number? Is there any even prime number?

Prime factorization or integer **factorization** of a number is the determination of the set of **prime** numbers which multiply together to give the original integer. It is also known as **prime** decomposition.

Prime factorization process:

Prime factors of 168 are $2, 2, 2, 3, 7$ and prime factors of 180 are $2, 2, 3, 3, 5$,

$$\begin{array}{r|l} 168 & 2 \\ 84 & 2 \\ 42 & 2 \\ 21 & 3 \\ 7 & 7 \\ 1 & \end{array} \qquad \begin{array}{r|l} 180 & 2 \\ 90 & 2 \\ 45 & 3 \\ 15 & 3 \\ 5 & 5 \\ 1 & \end{array}$$

$$2 \times 2 \times 2 \times 3 \times 7 = 168; \quad 2 \times 2 \times 3 \times 3 \times 5 = 180$$

Eratosthenes proposed a simple algorithm for finding prime numbers. This algorithm is known in mathematics as the Sieve of Eratosthenes.

In mathematics, the sieve of Eratosthenes, one of a number of prime number sieves, is a simple, ancient algorithm for finding all prime numbers up to any given limit. It does so by iteratively marking as composite, *i.e.*, not prime, the multiples of each prime, starting with the multiples of 2.

~~1~~	2	3	~~4~~	5	~~6~~	7	~~8~~	~~9~~	~~10~~
11	~~12~~	13	~~14~~	~~15~~	~~16~~	17	~~18~~	19	~~20~~
~~21~~	~~22~~	23	~~24~~	~~25~~	~~26~~	~~27~~	~~28~~	29	~~30~~
31	~~32~~	~~33~~	~~34~~	~~35~~	~~36~~	37	~~38~~	~~39~~	~~40~~
41	~~42~~	43	~~44~~	~~45~~	~~46~~	47	~~48~~	~~49~~	~~50~~
~~51~~	~~52~~	53	~~54~~	~~55~~	~~56~~	~~57~~	~~58~~	59	~~60~~
61	~~62~~	~~63~~	~~64~~	~~65~~	~~66~~	67	~~68~~	~~69~~	~~70~~
71	~~72~~	73	~~74~~	~~75~~	~~76~~	~~77~~	~~78~~	79	~~80~~
~~81~~	~~82~~	83	~~84~~	~~85~~	~~86~~	~~87~~	~~88~~	89	~~90~~
~~91~~	~~92~~	~~93~~	~~94~~	~~95~~	~~96~~	97	~~98~~	~~99~~	~~100~~

1. Proof that the sum of two any even natural numbers is an even number.
2. The remainder of 1932 ÷ 17 is 11, the remainder of 261 ÷ 17 is 6. Is 2193 = 1932 + 261 divisible by 17? Can you tell without calculating? Explain.
3. Find all natural numbers such that upon division by 7 the quotient and remainder will be equal.
4. Even or odd number will be the sum and the product of
   a. 2 odd numbers
   b. 2 even numbers
   c. 1 even and 1 odd number
   d. 1 odd and 1 even number

   Can you explain why? (a few examples do not prove the statement).
5. Compute (what is the best way to compute it?):
   a. $23 \times 15 + 15 \times 77;$
   b. $79 \times 21 - 69 \times 21;$

c. $340 \times 7 + 16 \times 70;$

d. $250 \times 61 - 25 \times 390;$

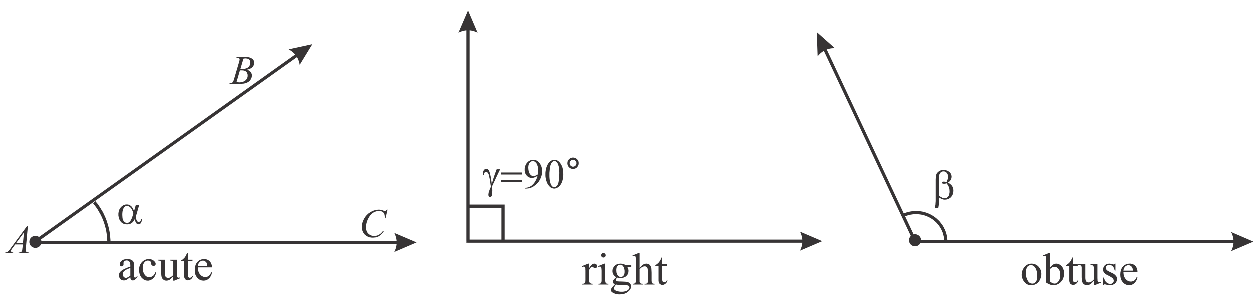
e. $67 \times 58 + 33 \times 58;$

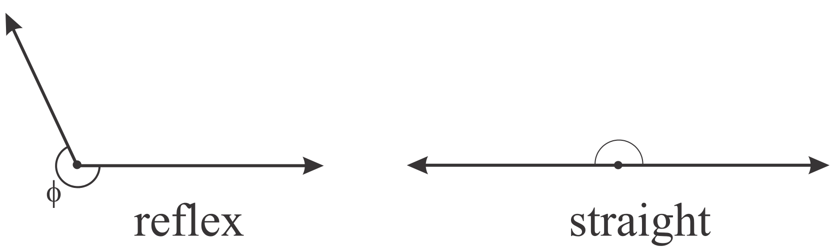
f. $55 \times 682 - 45 \times 682;$

Geometry.

An angle is the figure formed by two **rays**, called the sides of the angle, sharing a common endpoint, called the **vertex** of the angle.

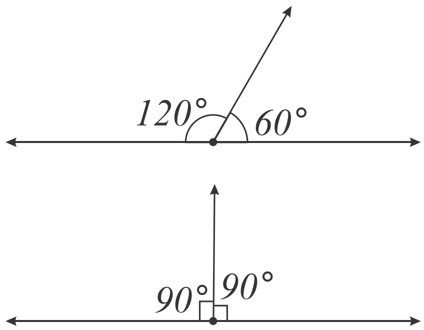
Angles notations are usually three capital letters with vertex letter in the middle or small Greek letter: $\angle ABC$, α. Measure of the angle is the amount of rotation required to move one side of the angle onto the other. As the angle increases, the name changes:



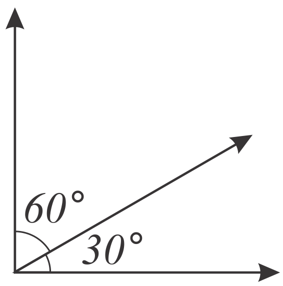


Strait angle is formed by two rays on the same strait line. Straight angle has a measure of 180°.

Two angles are called adjacent if they have common vertex and a common side. If two adjacent angles combined form straight angle they are called supplementary; if they form right angle than they are called complementary.



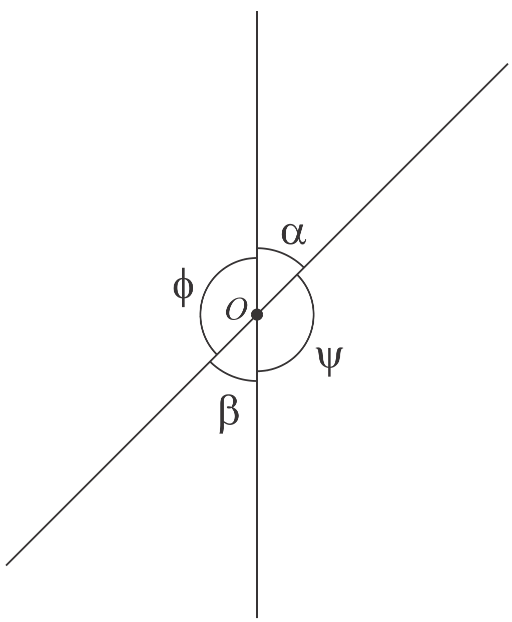
supplementary



complementary

An angle which is supplementary to itself we call right angle. Lines which intersect with the right angle we call perpendicular to each other.

When two straight lines intersect at a point, four angles are formed. A pair of angles opposite each other formed by two intersecting straight lines that form an "X"-like shape, are called vertical angles, or opposite angles, or vertically opposite angles.



α and β and ϕ and ψ are 2 pairs of vertical angles.

Vertical angles theorem:

Vertical angles are equal.

In mathematics, a **theorem** is a statement that has been proven on the basis of previously established statements.

According to a historical legend, when Thales visited Egypt, he observed that whenever the Egyptians drew two intersecting lines, they would measure the vertical angles to make sure that they were equal. Thales concluded that one could prove that vertical angles are always equal and there is no need to measure them every time.

Proof:

$\angle\phi + \angle\alpha = 180°$ because they are supplementary by construction.

$\angle\phi + \angle\beta = 180°$ because they are supplementary also by construction.

1. 4 angles are formed at the intersection of 2 lines. One of them is $30°$. What is the measure of 3 others?
2. 3 lines intersect at 1 point and form 6 angles. One is $44°$, another is $38°$. Can you find all other angles?