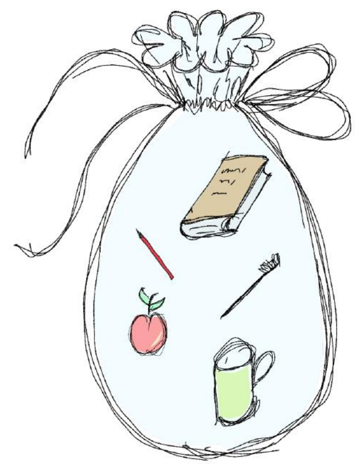
Math 4 d. Class work 2.

Sets.

I put a pencil, a book, a toothbrush, a coffee mug, and an apple into a bag. Do all these objects have something in common?



A set is a collection of objects that have something in common.

Can we call this collection of items a set? What is a common feature of all these objects? They are all in the bag, where I put them.

There are two ways of describing, or specifying, the members of a set. One way is by listing each member of the set, as we did with our set of things.

I can create, for example, a set of several flowers (F):

$F = \{rose,\ tulip,\ daffodil\}$.

The name of the set is usually indicated by a capital letter, in my case is F, list of members of the set is included in curved brackets. I can also create a set of all flowers, (N):

$$N = \{n |\ n = flower\} \quad (1)$$

Of cause, I can't itemize all possible flowers, there are too many of them, we don't have enough space here for that, but I can describe the common feature of all members of the set – they are all flowers. In the mathematical phrase above (1) I described this set, I call it N, also the set contains an unknown number of items, (I really don't know how many various flowers exist). I use variable n to represent these items, each of which is a flower. I show it by the phrase $n = flower$.

Exercise:

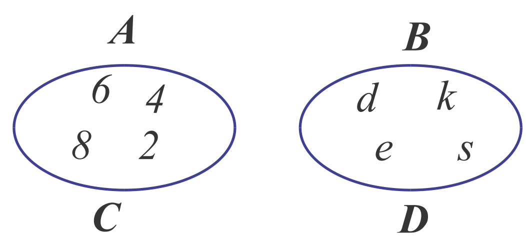
1. Give examples of several members of the following sets:

    Example:

    $$M = \{x |\ x = mammals\}$$

    x can be a lion, a whale, a bat…

    a. $K = \{y |\ y = letter\ of\ english\ alfabet\}$
    b. $M = \{x |\ x = animal\}$
    c. $X = \{m |\ m = even\ number\}$
    d. $P = \{k |\ k = color\}$



A = {2, 4, 6, 8}
B = {d, e, s, k}.

The second way is by using a rule:

Venn diagram.

C is the set of four first even natural numbers.
D is the set of letters of the word "desk".

Two sets are equal if they contain exactly the same elements. If we look closer on our sets ***A*** and ***C*** we can see that all elements of set ***A*** are the same as elements of set ***C*** (same goes for sets ***B*** and ***D***).

A*=*C and ***B*=*D***

If set ***A*** contains element '*2*', then we can tell that element '*2*' belongs to set ***A***. We have a special symbol to write it down in a shorter way: $2 \in A$, $105 \notin A$. (What does this statement mean?)

Let's define several sets.

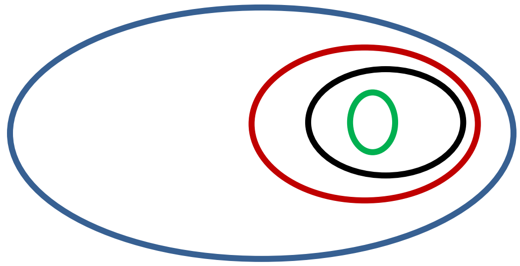
Set ***W*** will be the set of all words of the English language.

Set ***N*** will be the set of all nouns existing in the English language.

Set ***Z*** will be the set of all English nouns which have only 5 letters.

Set ***T***={"table"}. On a Venn diagram below name all these sets:

If all elements of one set at the same time belong to another set then we can say that the first set is a subset of the second one. We have another special symbol to write this statement in a shorter way: $\subset$.



$$\boldsymbol{T} \subset \boldsymbol{Z} \subset \boldsymbol{Y} \subset \boldsymbol{W}$$

If set ***V*** is defined as a set of all English verbs, can you draw a diagram for set ***V*** on the picture above? Can you tell subset of which set ***V*** is?

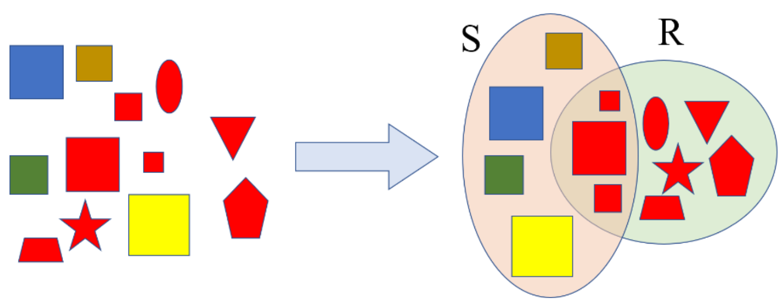
$\boldsymbol{V} \subset$ _____

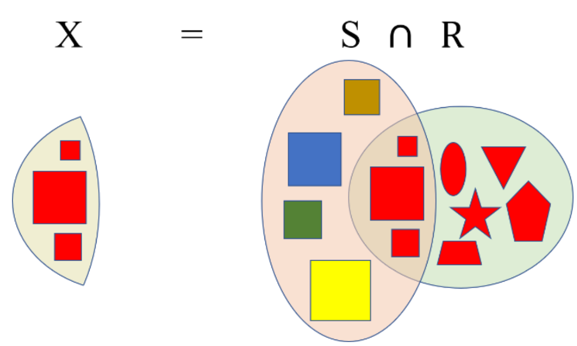
$\boldsymbol{V} \not\subset$ _____

$\boldsymbol{V} \not\subset$ _____

$\boldsymbol{V} \not\subset$ _____

When several sets are defined it can happen that in accordance with all the rules we have implied several objects can belong to several sets at the same time. For example, on a picture set ***S*** is a set of squares and a set ***R*** is a set of red figures. A few figures are squares and they are red, therefore they belong to both sets. Thus we can describe a new set ***X*** containing elements that belong to the set ***S*** as well as to the set ***R***. The new set was constructed by determining which



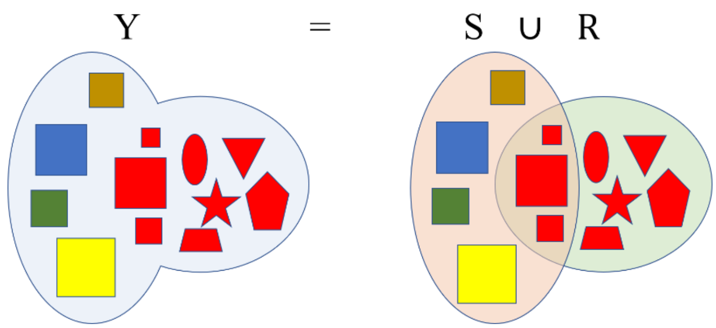


members of two sets have the features of both sets. This statement also can be written down in a shorter version by using a special symbol ∩. Such set $\boldsymbol{X}$ is called an **intersection** of sets $\boldsymbol{S}$ and $\boldsymbol{R}$.

$$\boldsymbol{X} = \boldsymbol{S} \cap \boldsymbol{R}$$

Another new set can be created by combining all elements of either sets (in our case $\boldsymbol{S}$ and $\boldsymbol{R}$). Using symbol ∪ we can easily write the sentence: Set $\boldsymbol{Y}$ contains all elements of set $\boldsymbol{S}$ and set $\boldsymbol{R}$:

$$\boldsymbol{Y} = \boldsymbol{S} \cup \boldsymbol{R}$$

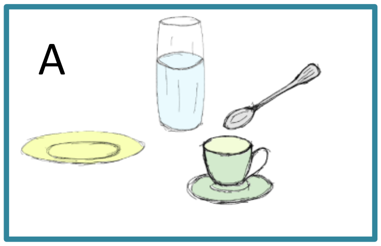


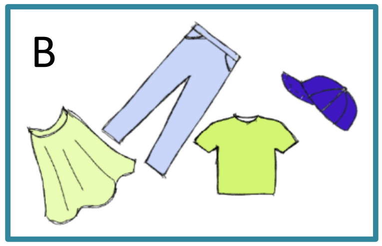
Such set $\boldsymbol{Y}$ is called a **union** of set $\boldsymbol{S}$ and $\boldsymbol{R}$. Set which does not have any element is called an empty set, in math people use symbol ∅).

∈	element belongs to a set
∉	element does not belong to a set
⊂	one set is a subset of another set
⊄	one set is not a subset of another set
∩	intersection of two sets
∪	union of two sets
∅	empty set

Exercises:

1. $A = \{a, b, c\}, B = \{1, 2, 3, 4\}$. Write the intersection $(A \cap B)$ and the union $(A \cup B)$ of these two sets.
2. Which word we can use to describe a set, subset of which is drawn on the pictures below:





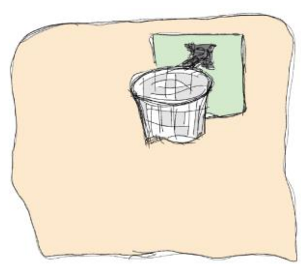
List a few other members of these sets.

3. There are 20 students in a Math class. 10 students like apples and 15 students like pears. Show that there are some students who like both apples and pears.

Assume that each student likes at least one of the fruits. (This means that each student like either apples, or pears, or both). How many students like both pears and apples? Is it possible to determine if there any students who do not like apples and do not like pears? If yes, explain how you can do it. If no, demonstrate by giving examples.

4. The same Math class (with 20 students) forms a soccer team and a basketball team. Every student signs up for at least one team:

- 12 students play only soccer;
- 2 students play both soccer and basketball;



How many students play basketball only?

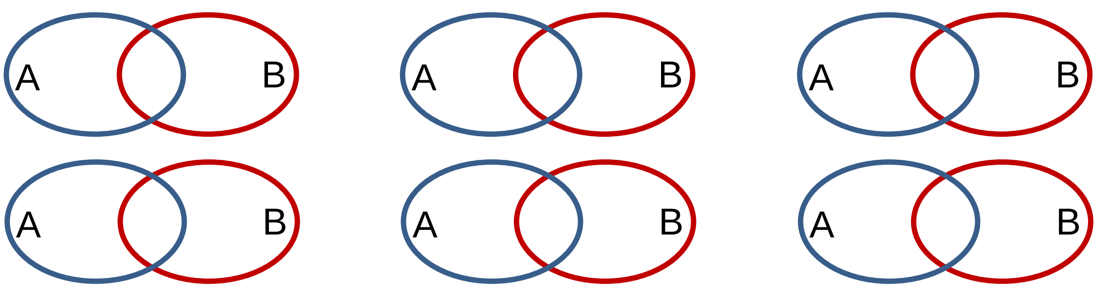
5. Students who participated in math coopetition had to solve 2 problems, one in algebra and another in geometry. Among 100 students 65 solved algebra problem, 45 solved geometry problem, 20 students solved both problems. How many students didn't solve any problem at all?

6. 240 students from New-York and Seattle attended a math camp. Of the total number of students, 125 were boys. 65 boys were from New-York. There were 53 girls from Seattle. How many students came from New-York?

7. In 2 boxes there are 160 notebooks altogether. In one box there are 20 more notebooks than in the other. How many notebooks are there in each box?

8. On the diagrams of sets A, and B put 4 elements so that (just draw 2 points, or put any two letters).

   a. each set contains 3 elements
   b. set A contains 2 elements, set B contains 4,
   c. set A contains 4 elements, sets B contains 3 elements,
   d. set A contains 0 elements, set B contains 4 elements,
   e. each set contains 2 elements,
   f. each set contains 4 elements.



9. John came to a lemonade stand with a big empty pitcher which can hold 5 liters of lemonade. He wanted to buy only 1 liter of lemonade, but a merchant had jars which can hold 3 liters and 2 liters of liquid. How merchant can measure 1 l. of lemonade if jars do not have any marks on them? Next time when John came to the stand with exactly the same pitcher, the merchant had only 3l and 5l jars. Can he sell to John exactly 4 l of lemonade?

Geometry.

A **definition** is a statement of the meaning of a something (term, word, another statement).

desk
noun
noun: **desk**; plural noun: **desks**

1. a piece of furniture with a flat or sloped surface and typically with drawers, at which one can read, write, or do other work.

nonyms: iting table, bureau, escritoire, secretaire, rolltop desk, carrel, ›rkstation, worktable

- Music
  a position in an orchestra at which two players share a music stand.
  "an extra desk of first and second violins"
- a counter in a hotel, bank, or airport at which a customer may check in or obtain information.
  "the reception desk"

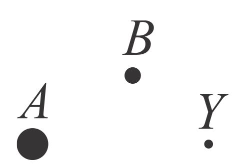
In mathematics everything (mmm,,,, almost everything) should be very well defined. In our real life it is also very useful and convenient to agree about terms and concepts, to give them a definition, before starting using them just to be sure that everybody knows what they are talking about. Now we move to geometry.

Can we give a definition to a point? Can we clearly define what a point is? What a line is? What a plane is?

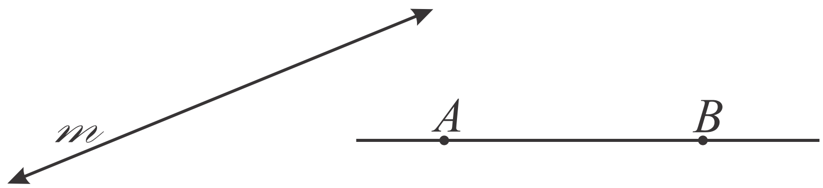
Mathematicians decided do not define terms "point", "straight line", and "plane" and to rely upon intuitive understanding of these terms.

Point (an undefined term).

In geometry, a point has no dimension (actual size), point is an exact location in space. Although we represent a point with a dot, the point has no length, width, or thickness. Our dot can be very tiny or very large and it still represents a point. A point is usually named with a capital letter.

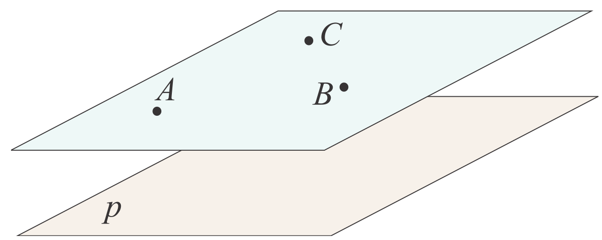


Line (an undefined term).



In geometry, a line has no thickness but its length extends in one dimension and goes on forever in both directions. Unless otherwise stated a line is drawn as a straight line with two arrowheads indicating that the line extends without end in both directions (or without them). A line is named by a single lowercase letter, *m* for example, or by any two points on the line, $\overleftrightarrow{AB}$ or AB.

Plane (an undefined term).



In geometry, a plane has no thickness but extends indefinitely in all directions. Planes are usually represented by a shape that looks like a parallelogram. Even though the diagram of a plane has edges, you must remember that the plane has no boundaries. A plane is named by a single letter (plane *p*) or by three non-collinear points (plane ABC).

A line segment is a part of a straight line between two chosen points.
(A set of points of a straight line between two points.)
These points are called endpoints.

A ray is a part of a straight line consisting of a point (endpoint) and all points of a straight line at one side of an endpoint. Ray is named by endpoint and any other point, ray $\overrightarrow{AB}$ or AB (where A is an endpoint)

Exercises:

1. Draw two line segments [AB] and [CD] in such way that their intersect
    a. by a point
    b. by a segment
    c. don't intersect at all.

2. Using a ruler draw a straight line, put on it 3 points, *A*, *B*, and *C* so that 2 rays are formed, *BC* and *BA*.
3. Draw two rays AB and CD in such way that their intersect
   d. by a point
   e. by a segment
   f. by a ray
   g. don't intersect at all