ADVANCED PHYSICS CLUB

OCTOBER 4, 2020

Useful resources

The updates, homework assignments, and useful links for APC can be found on SchoolNova's web page:
`https://schoolnova.org/nova/classinfo?class_id=adv_phy_club&sem_id=ay2020`
The practical information about the club and contacts can be found on the same web page.

Today's meeting

Today we are discussing hydrodynamics of an ideal fluid. Fluid being ideal means there is no viscosity, so energy is not dissipated throughout the motion. As a further simplification let us assume that fluid speed is constant in the direction transverse to motion, so the situation is essentially one-dimensional: everything depends only on the distance along the flow (this assumption actually only makes sense for an ideal fluid). Furthermore we are usually interested in a steady case when the fluid motion has begun a sufficiently long time ago so that nothing depends on time anymore.

The common procedure is then to consider some particular cross sections of the fluid flow and use conservation laws to relate different quantities in these cross sections. Suppose in the first cross section fluid density is ρ_1, fluid velocity is v_1, height above the ground is h_1, pressure is P_1 and the area of cross section is S_1. Correspondingly in the second cross section we have $\rho_2, v_2, h_2, P_2, S_2$. There are two relevant conservation laws : mass conservation and energy conservation. Mass conservation law implies:

$$\rho_1 v_1 S_1 = \rho_2 S_2 v_2 \,. \tag{1}$$

Varying density is really more characteristic to gases than to liquids. From now on let us concentrate on liquids and assume $\rho_1 = \rho_2 = \rho = const$. Then mass conservation law is just

$$v_1 S_1 = v_2 S_2 \,. \tag{2}$$

Energy conservation law implies in this case:

$$P_1 + \rho g h_1 + \frac{\rho v_1^2}{2} = P_2 + \rho g h_2 + \frac{\rho v_2^2}{2} \,. \tag{3}$$

The last formula is called Bernoulli law. One could see how it implies $P(h) = P_0 + \rho g h$ is the static (velocities are 0) case.

A particularly useful example of applying Bernoulli law refers to the liquid flowing out of some vessel. Imagine a big vessel full of water with a small hole near its' bottom. The water will flow through this hole. The question is what is the speed of water leaving the vessel. Choosing one cross section to be at the highest point of the liquid and the other to be just outside the hole and noting that at both of these points pressure in the water is just equal to the atmospheric pressure we obtain:

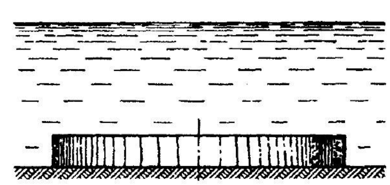
$$v = \sqrt{2gH} \,, \tag{4}$$

where H is the height of water level in the vessel measured from the hole. This is called Torricelli's equation.

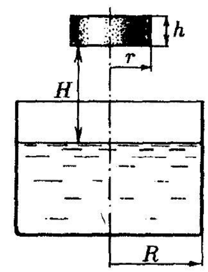
Problems

1. Try to derive Bernoulli Equation (3) and Torricelli's law (4) from the conservation of mechanical energy of the fluid.

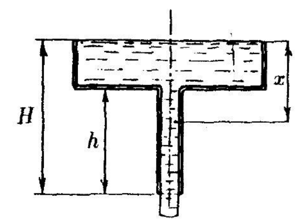
2. (**This is not a hydrodynamics problem**) A wooden cylinder of radius 1 m and height 0.2 m is attached to the bottom of a tank of depth 1 m. After being released it goes up. What quantity of heat is released after the cylinder and the water stop moving? Wood density is $0.8 \cdot 10^3 \ kg/m^3$



3. (**This is not a hydrodynamics problem**) Cylindrical cork of radius r and height h falls into a cylindrical tank of radius R filled with water. Initial height of the cork above the surface of the water is H and its' initial speed is zero. What quantity of heat will be released by the time cork and water stop moving? Density of cork is ρ, density of water is $\rho_0 > \rho$.

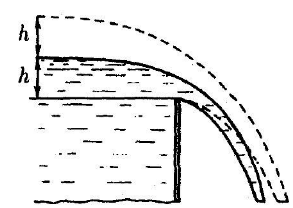


4. Liquid of density ρ flows out of a wide tank through a narrow tube in its' bottom. How do pressure and speed of the fluid depend on the vertical coordinate x? Atmospheric pressure is P_0, dimensions of the tank and the tube are shown on the picture.



5. A wide stream of water flows down a long inclined plane. Depth of the stream becomes two times smaller at a distance l down the flow. At what distance does depth become four times smaller?

***6.** Imagine a wide dam with water level being h above the edge. By how many times would the water discharge grow if the water level was $2h$ above the edge?



For the next meeting

IMPORTANT: The next club's meeting is at 3:00pm, via Zoom, on Sunday, **October 18**.