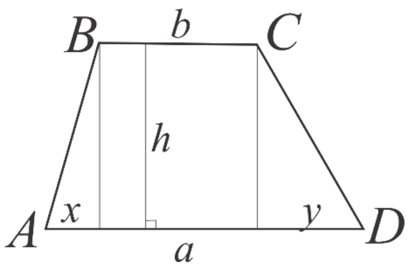
Classwork 25.

Area.

Trapezoid and the area of trapezoid.

Trapezoid is a convex quadrilateral which has a pair of parallel sides. Isosceles trapezoid has nonparallel sides are equal. Can you find the area of trapezoid?



$$S = S_x + S_y + S_{rec} = \frac{1}{2}xh + \frac{1}{2}yh + (a - x - y)h$$
$$= h\left(\frac{1}{2}x + \frac{1}{2}y + a - x - y\right) = h\left(a - \frac{1}{2}x - \frac{1}{2}y\right)$$
$$= \frac{1}{2}h(2a - x - y) = \frac{1}{2}h(a + a - x - y) = \frac{a+b}{2}h$$

Area of a circle.

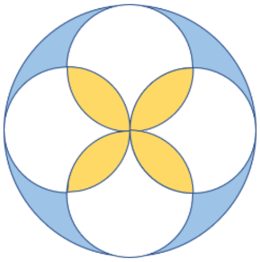
The ratio of circumference to the diameter is defined as π, the irrational number which can be rounded to 3.14.

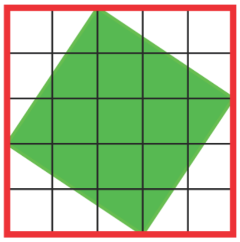
$$\frac{l}{2r} = \pi$$

Th area of the circle is

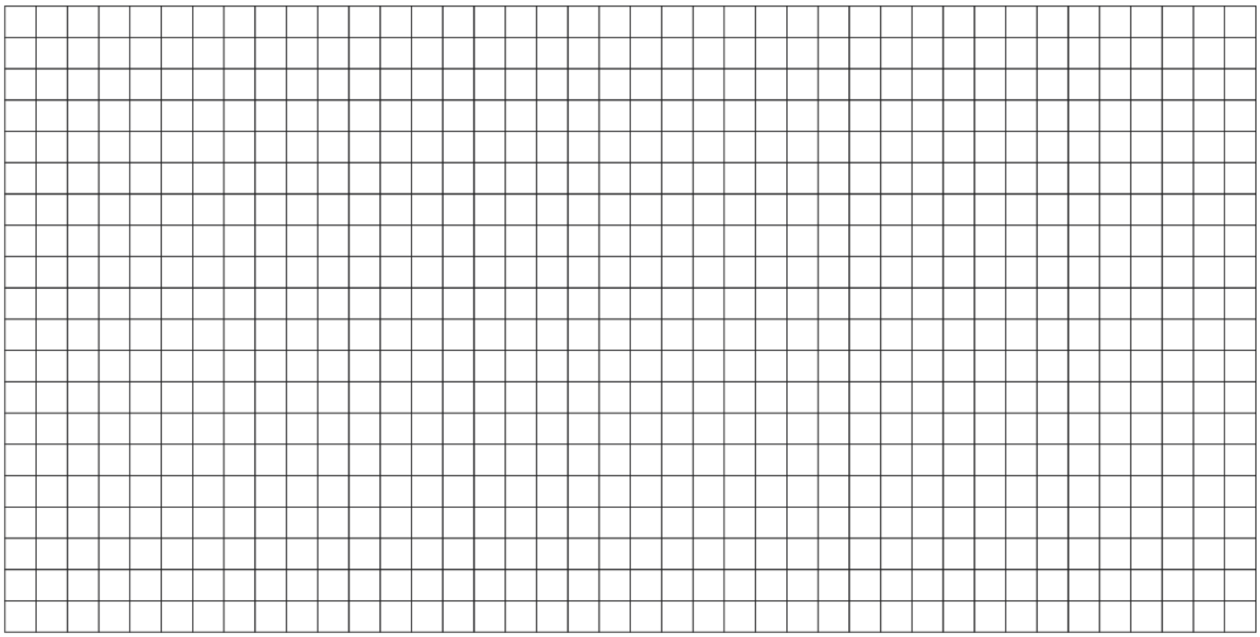
$$S = \pi r^2$$

1. How the area of a square will change if we increase the length of the side 2 times? 3 times? $2\frac{1}{2}$ times? How will change the area of a triangle if each of its side will be increase 2 times? 3 times?
2. How the area of the circle will change if the radius is increased two times? On the picture, the radius of the bigger circle is twice as big as the radius of the smaller circles.
3. Prove that the area shaded blue is the same as the area shaded yellow.
4. Prove that the area of the green square is 13 cm2 (assuming that the grid is 1 cm in each dimension).

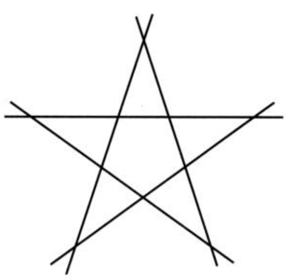


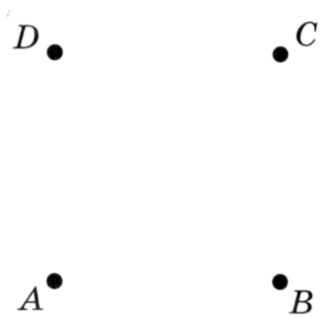


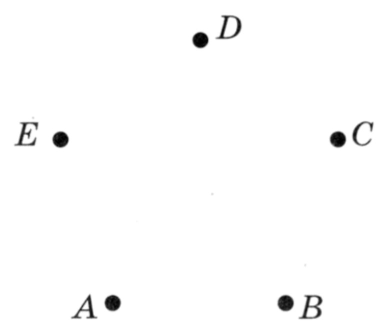
5. On a graph paper draw a square with the area equal to 2 cells, 4 cells, 5, 8, 9, 10, 16, 20, 35 cells.



6. How many lines are on the picture? How many lines can be drawn through 4 points? 5 points?







Quadratic equation.

Solve the equation:

$x^2 = 25,$ $y^2 = 7;$ $x^2 - 16 = 0;$ $x^2 + 4x + 4 = 0$

First two equations seem to be easy, we have to find a number, which square is 25 (these numbers are 5 and -5) and 7 (solutions are $\sqrt{7}$ and $-\sqrt{7}$).. the third equation can be modified to equivalent one $x^2 = 16$, or we can factorize it:

$$x^2 - 16 = x^2 - 4^2 = (x-4)(x+4) = 0$$

This equation will be equal to 0 when either of the factors is 0.

$$x - 4 = 0, \quad x + 4 = 0, \quad x = \pm 4$$

The third equation also can be factorizing:

$$x^2 + 4x + 4 = x^2 + 2 \cdot 2x + 2^2 = (x+2)^2 = (x+2)(x+2) = 0$$

We used the identity $(a+b)^2 = a^2 + 2ab + b^2$; but we can just use the distributive property:

$$x^2 + 4x + 4 = x^2 + 2x + 2x + 4 = x(x+2) + 2(x+2) = (x+2)(x+2)$$

This equation has only one root, $x = -2$.

This procedure is called completing the full square. Let's see haw it works for more complicated equations, for example $x^2 + 6x + 8 = 0$

$$x^2 + 6x + 8 = x^2 + 2 \cdot 3 \cdot x + 8 = x^2 + 2 \cdot 3 \cdot x + 9 - 9 + 8 = (x+3)^2 - 1 = (x+3)^2 - 1^2$$
$$= (x+3+1)(x+3-1) = (x+4)(x+2) = 0$$

We used the identity $a^2 - b^2 = (a-b)(a+b)$. But ale we can factorize the polynomial directly:

$$x^2 + 6x + 8 = x^2 + 2x + 4x + 8 = x(x+2) + 4(x+2) = (x+2)(x+4) = 0$$

The equation is equal to 0, if any of the factors is 0, so the roots are -4, and -2.

Let's try to solve another equation:

$$2x^2 + 7x + 30 = 0, \ \Rightarrow x^2 + \frac{8}{2}x + \frac{30}{2} = 0; \ \Rightarrow x^2 + 2 \cdot \frac{4}{2}x + \frac{30}{2} = x^2 + 2 \cdot 2 \cdot x + 15$$
$$= x^2 + 2 \cdot 2 \cdot x + 4 - 4 + 15 = (x+2)^2 + 15 = 0$$

Does this equation have any roots?

$$3x^2 + 2x - 10 = 0; \quad x^2 + 2 \cdot \frac{1}{3} \cdot x - 10 = 0;$$

$$x^2 + 2 \cdot \frac{1}{3} \cdot x - 10 = x^2 + 2 \cdot \frac{1}{3} \cdot x + \frac{1}{9} - \frac{1}{9} - 10 = \left(x + \frac{1}{3}\right)^2 - \frac{1}{3} - 10 = \left(x + \frac{1}{3}\right)^2 - \frac{31}{3}$$

$$= \left(x + \frac{1}{3}\right)^2 - \left(\sqrt{\frac{31}{3}}\right)^2 = \left(x + \frac{1}{3} + \sqrt{\frac{31}{3}}\right)\left(x + \frac{1}{3} - \sqrt{\frac{31}{3}}\right) = 0$$

$$x_1 - \frac{1}{3} - \sqrt{\frac{31}{3}}; \qquad x_2 = -\frac{1}{3} + \sqrt{\frac{31}{3}}$$

Can we complete square in general form and find the general form of the root of quadratic equation?

Polynomial of the second order can be written generally as:

$$ax^2 + bx + c$$

a, b, c are real numbers and x is a variable.
First, let's move the common factor (fist coefficient) out:

$$ax^2 + bx + c = a\left(\frac{a}{a}x^2 + \frac{b}{a}x + \frac{c}{a}\right) = a\left(x^2 + \frac{b}{a}x + \frac{c}{a}\right)$$

Do you remember the algebraic identity

$$(k + z)^2 = (k + z)(k + z) = k^2 + 2kz + z^2$$

The last expression is

$$\left(x^2 + \frac{b}{a}x + \frac{c}{a}\right)$$

Looks like $k^2 + 2kz + z^2$.

$$\left(x^2 + \frac{b}{a}x + \frac{c}{a}\right) = \left(x^2 + 2 \cdot \frac{1}{2}\frac{b}{a}x + \left(\frac{b}{2a}\right)^2 - \left(\frac{b}{2a}\right)^2 + \frac{c}{a}\right)$$

$$= \left(x^2 + 2 \cdot \frac{b}{2a}x + \left(\frac{b}{2a}\right)^2\right) - \left(\left(\frac{b}{2a}\right)^2 - \frac{c}{a}\right)$$

We can further transform the expression:

$$\left(x^2 + 2 \cdot \frac{b}{2a}x + \left(\frac{b}{2a}\right)^2\right) - \left(\left(\frac{b}{2a}\right)^2 - \frac{c}{a}\right) = \left(x + \frac{b}{2a}\right)^2 - \left(\sqrt{\left(\frac{b}{2a}\right)^2 - \frac{c}{a}}\right)^2 =$$

Last part is actually another identity:

$$p^2 - q^2 = (p + q)(p - q):$$

Therefore, we can rewrite it as

$$\left(x+\frac{b}{2a}\right)^2-\left(\sqrt{\frac{b^2}{4a^2}-\frac{c}{a}}\right)^2=\left(x+\frac{b}{2a}\right)^2-\left(\sqrt{\frac{b^2}{4a^2}-\frac{4ac}{4a^2}}\right)^2=\left(x+\frac{b}{2a}\right)^2-\left(\sqrt{\frac{b^2-4ac}{4a^2}}\right)^2$$

$$=\left(x+\frac{b}{2a}-\frac{\sqrt{b^2-4ac}}{2a}\right)\left(x+\frac{b}{2a}+\frac{\sqrt{b^2-4ac}}{2a}\right)$$

Polynomial of the second order can be factorized as

$$ax^2+bx+c=a\left(x+\frac{b}{2a}-\frac{\sqrt{b^2-4ac}}{2a}\right)\left(x+\frac{b}{2a}+\frac{\sqrt{b^2-4ac}}{2a}\right)$$

a, b, c are numbers. Do you think any quadratic polynomial can be factorized?
Can we use this factorization process to find the way to solve a quadratic equation? Quadratic polynomial is an expression in the form
$P(x)=ax^2+bx+c$
$P(x)$ means that the value of the expression depends of the value of x. What if we want to find such x that the polynomial $P(x)$ will have a certain value? For example, which x will bring the polynomial (I will omit the word “quadratic” now, but we remember, that we are talking about the quadratic polynomials) $x^2-6x+13$ to 5. In other words, can we solve the equation:

$$x^2-6x+13=5$$

First, let’s rewrite this equation as

$$x^2-6x+13-5=0$$
$$x^2-6x+8=0$$

Such equations we are going to call the quadratic equation.
We have just proved that

$$ax^2+bx+c=a\left(x+\frac{b}{2a}-\frac{\sqrt{b^2-4ac}}{2a}\right)\left(x+\frac{b}{2a}+\frac{\sqrt{b^2-4ac}}{2a}\right)$$

For any $a, b,$ and c, $a \neq 0$. If $a=0$, the polynomial $P(x)$ is not quadratic.

$$a\left(x+\frac{b}{2a}-\frac{\sqrt{b^2-4ac}}{2a}\right)\left(x+\frac{b}{2a}+\frac{\sqrt{b^2-4ac}}{2a}\right)$$

Can be equal to 0 only if either of the expressions inside of the parenthesis is equal to 0.
If

$$x+\frac{b}{2a}-\frac{\sqrt{b^2-4ac}}{2a}=0; \quad x+\frac{b}{2a}+\frac{\sqrt{b^2-4ac}}{2a}=0$$

$$x_1=-\left(\frac{b}{2a}-\frac{\sqrt{b^2-4ac}}{2a}\right)=\frac{-b+\sqrt{b^2-4ac}}{2a}$$

$$x_2=-\left(\frac{b}{2a}+\frac{\sqrt{b^2-4ac}}{2a}\right)=\frac{-b-\sqrt{b^2-4ac}}{2a}$$

We got two very similar expressions for our solutions:

$$x_1 = \frac{-b + \sqrt{b^2 - 4ac}}{2a} \quad \text{and} \quad x_2 = \frac{-b - \sqrt{b^2 - 4ac}}{2a}$$

The expression $b^2 - 4ac$ is called the discriminant, and denoted as D.

$$D = b^2 - 4ac$$

For each quadratic polynomial we can find $D = b^2 - 4ac$. D is under the radical sign ($\sqrt{}$) in the expressions for the roots of quadratic equation, so D can't be less than 0.

1. If $D > 0$, the equation has different roots
2. If $D = 0$, the second part of the expressions for x becomes 0, and both roots are the same, $-\frac{b}{2a}$, and polynomial is a full square,

$$ax^2 + bx + c = a\left(x + \frac{b}{2a}\right)\left(x + \frac{b}{2a}\right) = a\left(x + \frac{b}{2a}\right)^2$$

3. If $D < 0$, then the square root of a negative number can't be a real number and the equation doesn't have any solution among the real numbers (but has solutions in a set of complex numbers; complex numbers are outside of the scope of our class, you will learn about them later.)

We can also rewrite the expression for factorization of the quadratic polynomial using the discriminant:

$$P(x) = ax^2 + bx + c = a\left(x + \frac{b}{2a} - \frac{\sqrt{b^2 - 4ac}}{2a}\right)\left(x + \frac{b}{2a} + \frac{\sqrt{b^2 - 4ac}}{2a}\right)$$

$$= a\left(x + \frac{b - \sqrt{D}}{2a}\right)\left(x + \frac{b + \sqrt{D}}{2a}\right)$$