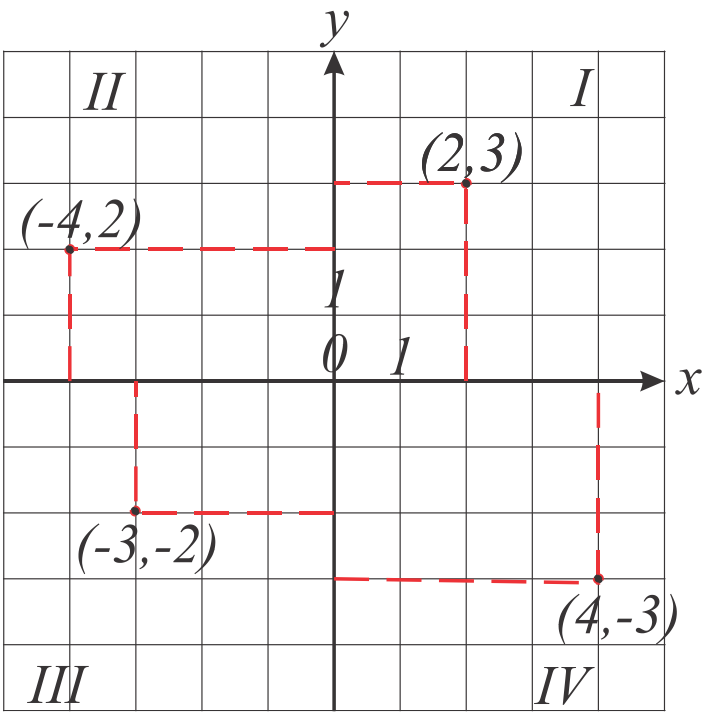
Classwork 21. Algebra.

Algebra.

Cartesian coordinate system.

On a number line each point represents a number and each number is linked to a point if an origin (point at 0) and a unit segment are defined. This number is a coordinate of a point on the line in the defined coordinate system: absolute value of this number shows the distance (how many unit segments can be put in) between the point and the origin and the sign shows on which side of the origin this point is located. On a plane each point corresponds to a unique ordered pair of numbers. To define this pair for each point 2 perpendicular number line are usually used. These two number lines intersect at the point called origin, associated with pair (0,0), have the same unit segment, and are called axes, usually x axis and y axis. The pair of numbers allied with each point of the plane in this particular system of coordinate defined as a distance from the point to both axes, and the signs of these numbers correspond to a quadrant where point is located (quadrants I, II, III, and IV on the picture above). Such pair of numbers is an ordered pair, so the pair (n,m) and the pair (m,n) are linked to two different points. Absolute value of the first number in the pair is the distance from the point to the y axis and absolute value of the second one is the distance from the point to the x axis.



Function.

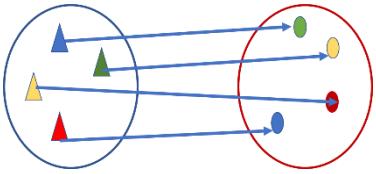
We know that the area of the square is the second power of its side. It can be written as

$$S = a^2, \quad a > 0$$

For any value of a there is a single value of S, that can be found with the help of set of rules, described in a function. In the previous example, the rule is "multiply the value of a by itself to get the value of S). Such relationship between two variables is called function. Function can be thought as a box where you put your input and then get output from. Can you find the rule for:

x	2	3	4	5	6	7
y	6	9	12	15	18	?

This imaginary box can produce only one output for each input, but output can be the same for different inputs. More formal definition of the function is



Function is a rule which connect each element of one set with only one element of the other set.

The set of possible inputs is called the **domain**, and the set of possible outputs is called the **range** of the function. In algebra, we are going to work with sets of numbers, and our functions will relate one set of numbers with another.

What is domain of the function in our table?

In the example of the area of a square S(area) is a function of a (side), thise also can be written as

$$S = f(a); \quad f(a) = a^2, \quad a > 0$$

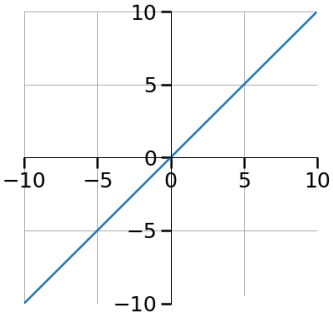
The functions:

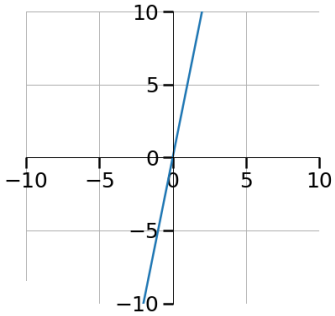
$y = 5, \quad y = x, \quad f(m) = m^2, \quad g(y) = 2^y$

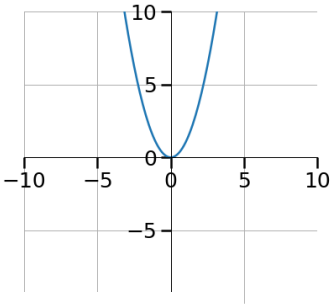
But usually, x is reserved for an independent variable and y for the variable which depend upon x. For example, for the function $f(x) = x^2$, x Is independent variable and for each possible value of x, value of $y = f(x) = x^2$ can be calculated. :

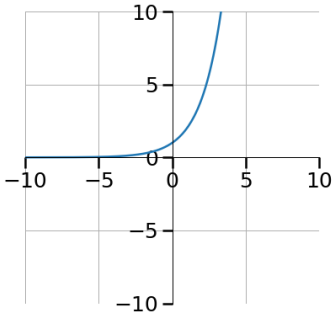
$f(1) = 1, \quad f(2) = 4, \quad f(3) = 9, \quad f(-2) = 4$

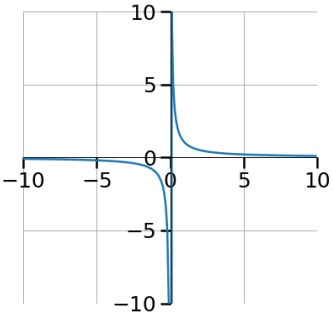
When there is a function, there are set of pairs of numbers, in each such pair the first number is the value of independent variable (input) and the second number is the corresponding value of function. These pairs can be the coordinates of a point on a coordinate plane.











Above are the plots of functions $y = 5x, y = x, y = \frac{1}{x};\ y = x^2, y = 2^x$

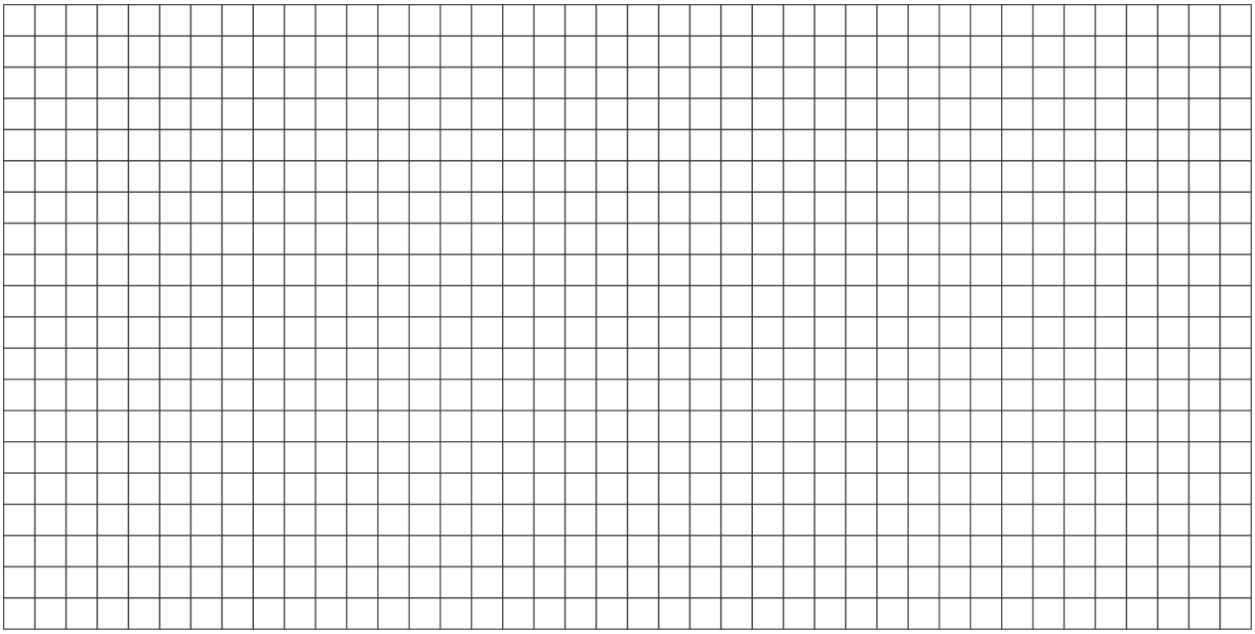
Can you defined which graph represent which function?

Let's explore a function $y = ax + b$.

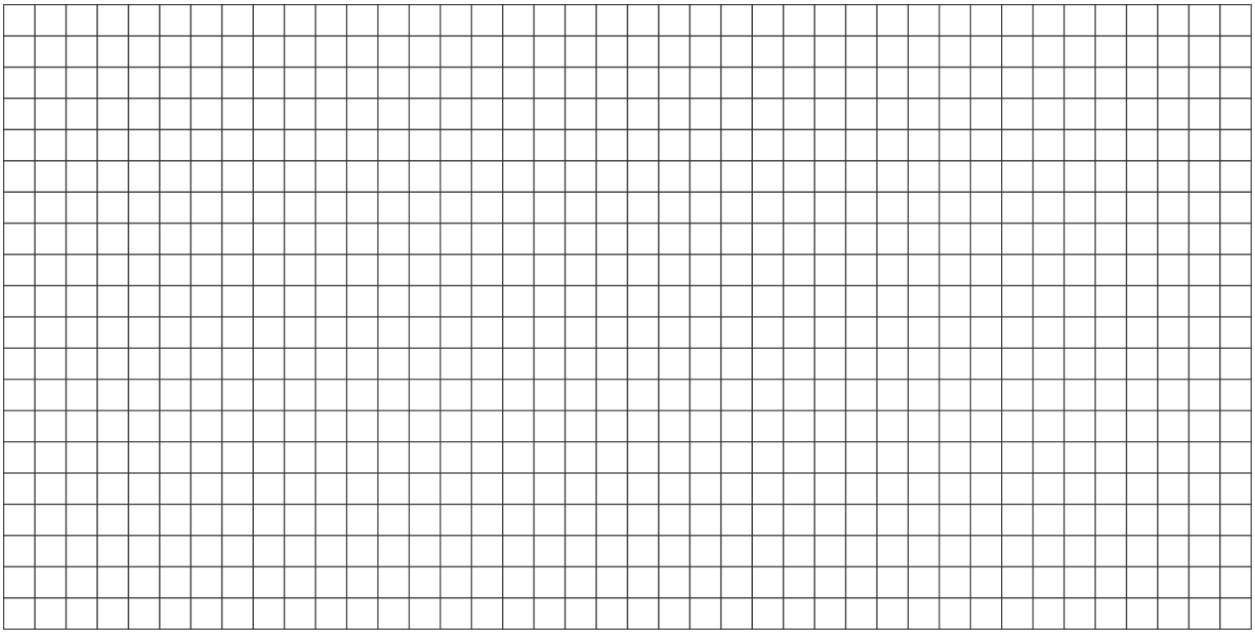
What is a domain? What is a range of this function?

Try to plot

$y = x,\ y = -x,$ $y = 2x,$ $y = \frac{1}{2}x$ How does the coefficient affect the line?



$y = x + 2;$ $y = x - 2,$ $y = 5.$ How does the constant term affect the line?



The line goes through the points (1, 4) and (3, 10). Write the equation of the (linear) function.
General equation of linear function is

$$y = ax + b$$

where x is an independent variable and y is a corresponding value of the function. If the faction is passing through point with coordinates (1, 4) it means, that when the value of x is 1, the value of y is 4.

$$4 = a + b$$

And

$$10 = 3a + b$$

The pair of linear equations, the solution of which is the pair (a,b).

$$\begin{cases} 4 = a + b \\ 10 = 3a + b \end{cases} \Rightarrow \begin{cases} a + b = 4 \\ 3a + b = 10 \end{cases}; \qquad b = 4 - a, \quad 3a + 4 - a = 10, \quad 2a = 6, \quad a = 3$$

$$b = 4 - a = 4 - 3 = 1 \qquad y = 3x + 1$$

Function can be defined with table, with the formula, with the graph, by description.
Here is example of the function of the air temperature depending of time. The temperature is changing continually, without interruption, a small increase in time causes a small increase in temperature