Algebra and Geometry 1. Homework 20.

1. Make an equation of the first order ($ax + b = 0$) and solve them, if

   a. $a = -3, \quad b = 5$ *b.* $a = -\frac{1}{4}, \quad b = 7$

   c. $a = 2, \quad b = 0$ *d.* $a = \frac{1}{2}, \quad b = -10$

   e. $a = 30, \quad b = -20$ *f.* $a = 7\frac{1}{2}, \quad b = -8$

2. Is the pair of numbers (1;2) a solution of the system:

   a. $\begin{cases} x + y - 3 = 0 \\ x - y + 1 = 0 \end{cases}$ *b.* $\begin{cases} 2.5x - 2.5 = 0 \\ \frac{1}{4}y - \frac{1}{2} = 0 \end{cases}$

   c. $\begin{cases} 2x + 3y - 8 = 0 \\ 4x - y - 2 = 0 \end{cases}$ *d.* $\begin{cases} 0.35x + 1.6y - 3.55 = 0 \\ \frac{x}{6} - \frac{y}{7} + \frac{5}{42} = 0 \end{cases}$

3. Solve the systems:

   a. $\begin{cases} x + 2y - 3 = 0 \\ 2x - 3y + 8 = 0 \end{cases}$ *b.* $\begin{cases} 2x - y - 8 = 0 \\ 3x + 4y - 7 = 0 \end{cases}$

   c. $\begin{cases} -6x + 2y + 6 = 0 \\ 5x - y - 17 = 0 \end{cases}$ *d.* $\begin{cases} 5x + 3y - 7 = 0 \\ 2x - y - 5 = 0 \end{cases}$

4. Bronze is an alloy of tin and copper. (Tin and copper are metals; they are melted together to get an alloy which is called bronze). How much copper and how much tin are there in the 80 kg piece of bronze, if the ratio of tin to copper in bronze is 3 to 17?

5. Five ladies and five gentlemen gathered at the ball. How many ways do they have to form pairs to dance the waltz?
6. How many four-digit numbers there are:
   a) consisting only of odd digits;
   b) consisting only of even digits;
   c) with at least one odd digit;
   (0 is not even number, repetition of digits is allowed)
7. Simplify:
   Example:

$$\frac{a^{-3} + b^{-3}}{a^{-1} + b^{-1}} = \frac{\frac{1}{a^3} + \frac{1}{b^3}}{\frac{1}{a} + \frac{1}{b}} = \frac{\frac{b^3 + a^3}{a^3 b^2}}{\frac{a + b}{ab}} = \frac{b^3 + a^3}{a^3 b^2} \cdot \frac{ab}{a + b}$$

$$b^3 + a^3 = b^3 + b^2a - b^2a + a^2b - a^2b + a^3 = b^3 + b^2a - b^2a - a^2b + a^2b + a^3$$
$$= b^2(b + a) - ba(b + a) + a^2(b + a) = (b + a)(b^2 - ba + a^2)$$
$$= (a + b)(a^2 - ba + b^2)$$

$$\frac{b^3 + a^3}{a^3b^2} \cdot \frac{ab}{a + b} = \frac{(a + b)(a^2 - ba + b^2)}{a^3b^2} \cdot \frac{ab}{a + b} = \frac{(a^2 - ba + b^2)}{a^2b^2}$$

a. $\dfrac{a^{-2} - b^{-2}}{a^{-1} + b^{-1}}$

b. $\dfrac{a^{-3} - b^{-3}}{a^{-1} - b^{-1}}$