Classwork 20. Algebra.

Algebra.

Linear equations.

We used to solve equations, like this one.

$$3x + 5 = x - 1$$

We can rewrite it to be able to combine all like terms and add all numbers and solve:

$$3x - x = -1 - 5 = -6$$

$$2x = -6, \quad x = (-6) : 2 = -3$$

But also, we can write it as

$$2x + 6 = 0 \Leftrightarrow 2x = -6$$

Equations, similar to $2x + 6 = 0$ are called equations of the first order with one variable.

Expression $2x + 6$ is a polynomial of the first degree. Other examples:

$7x - 3 = 0, \quad -5x + 10 = 0, \qquad x - 1 = 0, \quad 4x = 0$

Equation that can written the form

$$ax + b = 0 \qquad\qquad (1)$$

where a, b are real numbers and x is unknown, $a \neq 0$ is a linear equation. a is a coefficient, and b is a constant term.

Suppose that the equation (1) has the solution x_0. It means that

$$ax_0 + b = 0 \; is\ true.$$

$$\text{and} \qquad x_0 = -\frac{b}{a}$$

So, if x_0 is a root of equation (1) it is equal to $-\frac{b}{a}$. To check it, let's substitute x_0 to the equation (1):

$$a\left(-\frac{b}{a}\right) + b = 0, \qquad -\frac{ba}{a} + b = -b + b = 0$$

We proved that the

$$x_0 = -\frac{b}{a}$$

is a single root of the equat0ion (1).

If $a = 0$ and $b \neq 0$, equation doesn't have any roots, if $a = 0$ and $b = 0$, equation has infinitely many roots, any number can be a solution,

A linear one-variable equation is an equation whose left and right sides are polynomials of no higher than first order.

Linear equation

$$ax + by + c = 0 \qquad (2)$$

Where $a, b, c,$ are real numbers, and at least one of a and b is not 0 is called an equation of first degree with two unknowns. Numbers a and b are coefficients and c is a constant term. The pairs of numbers $(x_{0,}\ y_0)$ is a solution of the equation (2) if these values make it a true statement.

For example

$$3x - 2y + 6 = 0$$

$a = 3, b = -2, c = 6.$ Pair (0,3) is the solution : $3 \cdot 0 - 2 \cdot 3 + 6 = 0$

There are infinitely many solutions of the equation (2). If we take $x_0 = 1,$ then

$$3 \cdot 1 - 2y_0 + 6 = 0, \quad 9 = 2y_{0,} \quad y_0 = \frac{9}{2}$$

For any x_0 we can find a corresponding y_0

$$3x_0 - 2y + 6 = 0 \quad \Rightarrow \quad 3x_0 + 6 = 2y \quad \Rightarrow \quad y = \frac{3x_0 + 6}{2} = \frac{3}{2}x_0 + 3$$

We always can find y_0 for any x_0.

$$ax_0 + by + c = 0 \Rightarrow \quad by = -c - ax_0, \quad y = -\frac{c + ax_0}{b}$$

We can have a set of two linear equations with two variable. If we need to solve both equations so that the one pair of variables make both equations true statements, we call these equations a system of two equations of the first order. (We can have more variable and more equations, as I will show later).

$$\begin{cases} a_1x + b_1y + c_1 = 0 \\ a_2x + b_2y + c_2 = 0 \end{cases}$$

To solve the system of two equations means to find a pair of numbers (x_0, y_0) which is a solution for both equations.

$$\begin{cases} 2x - 3y + 1 = 0, \\ x + 2y + 4 = 0; \end{cases}$$
$$\begin{cases} x - y + 1 = 0, \\ 2x - 2y + 3 = 0; \end{cases}$$
$$\begin{cases} 2x + y + 2 = 0, \\ 6x + 3y + 6 = 0; \end{cases}$$
$$\begin{cases} 3x + 0y + 1 = 0, \\ 2x + y - 5 = 0; \end{cases}$$
$$\begin{cases} 2x + 0y - 5 = 0, \\ 3x + 0y + 2 = 0; \end{cases}$$
$$\begin{cases} 5x + 0y - 1 = 0, \\ 0x + 3y + 2 = 0. \end{cases}$$

In the second and third systems, coefficients are proportional, $a_1 : a_2 = b_1 : b_2$.

There are few ways to solve systems with non-proportional nonzero coefficients. We will take a look on two of them: substitution and balancing coefficients. Example:

$$\begin{cases} 2x - y + 4 = 0 \\ 3x + 4y - 27 = 0 \end{cases} ; \quad (x_0, y_0) \text{ is a solution: } \begin{cases} 2x_0 - y_0 + 4 = 0 \\ 3x_0 + 4y_0 - 27 = 0 \end{cases}$$

From the first equation we can write: $y_0 = 2x_0 + 4$ and we can write this expression instead of y_0 into the second equation.

$$3x_0 + 4(2x_0 + 4) - 27 = 0$$

So, the number x_0 satisfy the equation $3x + 4(2x + 4) - 27 = 0$.

$$3x + 4(2x + 4) - 27 = 0,$$

$$3x + 8x + 16 - 27 = 0$$

$$11x - 11 = 0, \quad x_0 = 1$$

$$y_0 = 2 \cdot 1 + 4 = 6$$

Pair (1;6) is a solution of the system.

$$\begin{cases} 2x - y + 4 = 0 \\ 3x + 4y - 27 = 0 \end{cases} \quad \begin{matrix} \times 4 \\ \times 1 \end{matrix} \quad \begin{cases} 8x - 4y + 16 = 0 \\ 3x + 4y - 27 = 0 \end{cases}$$

Now we can add two equations from : $11x - 11 = 0, x = 1, y = 6.$

Exercises:

1. Make an equation of the first order and solve them, if

$a.\quad a = -3, \qquad b = 5$ $\qquad$ $b.\quad a = -\frac{1}{4}, \qquad b = 7$

$c.\quad a = 2, \qquad b = 0$ $\qquad$ $d.\quad a = \frac{1}{2}, \qquad b = -10$

$e.\quad a = 30, \qquad b = -20$ $\qquad$ $f.\quad a = 7\frac{1}{2}, \qquad b = -8$

$g.\quad a = 0.\overline{3}, \qquad b = 0$ $\qquad$ $b.\quad a = -7.5, \qquad b = 4$

2. Is the nimber $\frac{1}{2}$ a root of the equation

$a.\ 5x - 8 = 0$ $\qquad$ $b.\ 1.3x - 0.65 = 0$ $\qquad$ $c.\ 4x - 8 = 0$

$d.\ 7\frac{1}{4}x - 3.5 = 0$ $\qquad$ $e.\ 8x - 4 = 0$ $\qquad$ $f.\ \frac{1}{2}x = 0$

3. Is the pair of numbers (1;2) a solution of the system:

$a.\quad \begin{cases} x + y - 3 = 0 \\ x - y + 1 = 0 \end{cases}$ $\qquad$ $b.\quad \begin{cases} 2.5x - 2.5 = 0 \\ \frac{1}{4}y - \frac{1}{2} = 0 \end{cases}$

$c.\quad \begin{cases} 2x + 3y - 8 = 0 \\ 4x - y - 2 = 0 \end{cases}$ $\qquad$ $b.\quad \begin{cases} 0.35x + 1.6y - 3.55 = 0 \\ \frac{x}{6} - \frac{y}{7} + \frac{5}{42} = 0 \end{cases}$

4. Which values of a and b will make a pair of numbers (1;0) a solution of the sytem:

$a.\quad \begin{cases} 2x + y = a \\ bx - y = 2 \end{cases}$ $\qquad$ $b.\quad \begin{cases} 3x - ay = 3 \\ 2x + y = b \end{cases}$

5. Solve the system by substitution:

а) $\begin{cases} x - y - 1 = 0, \\ x + y - 5 = 0; \end{cases}$ $\qquad$ б) $\begin{cases} x - y - 2 = 0, \\ x + y - 6 = 0; \end{cases}$

в) $\begin{cases} x - y - 2 = 0, \\ 3x - 2y - 9 = 0; \end{cases}$ $\qquad$ г) $\begin{cases} x - 2y - 3 = 0, \\ 5x + y - 4 = 0; \end{cases}$

6. Solve y balance coefficients:

а) $\begin{cases} x + 2y - 3 = 0, \\ 2x - 3y + 8 = 0; \end{cases}$ $\qquad$ б) $\begin{cases} 2x + y - 8 = 0, \\ 3x + 4y - 7 = 0; \end{cases}$

в) $\begin{cases} -6x + 2y + 6 = 0, \\ 5x - y - 17 = 0; \end{cases}$ $\qquad$ г) $\begin{cases} 5x + 3y - 7 = 0, \\ 2x - y - 5 = 0; \end{cases}$