Classwork 18. Algebra.

Algebra.

Equalities: equations and identities

Inequalities.

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We already discussed the difference between equation and identity. Equality, which is true for any value of the variables included in it, is called identity. For example:

$$(x+y)^2 = x^2 + 2xy + y^2$$

Is an identity, it's true for any value of x and y. By doing identical transformation we can change one expression with another, creation an identity.

$$2(a+b) = 2a + 2b$$

Not everything we can do with an algebraic expression is an identical transformation, for example, the reducing the fraction is not always the identical transformation, even if it looks like one.

$$\frac{x^2+6x+9}{x+3} \underbrace{=}_{still\ identical} \frac{(x+3)^2}{x+3} \underbrace{=}_{not\ identical} x+3$$

Even we can divide both, nominator and denominator of the fraction by the same number or expression, these changes are not identical transformations, we can lose the information about the possible values of the variables. In our example, the right part $x+3$ is defined on the set of real numbers (x can be anything, from $-\infty$ to $+\infty$, but in the initial expression

$$\frac{x^2+6x+9}{x+3}$$

There is one value for x that makes the whole expression meaningless, -3.

$$\frac{x^2+6x+9}{x+3} = x+3, \quad x \neq -3$$

Another example of simplifying fraction:

$$\frac{x^2-1}{x^2+x-2} = \frac{(x+1)(x-1)}{x^2+x-1-1} = \frac{(x+1)(x-1)}{(x^2-1)+(x-1)} = \frac{(x+1)(x-1)}{(x+1)(x-1)+(x-1)}$$

$$= \frac{(x+1)(x-1)}{(x-1)(x+1+1)} = \frac{(x+1)(x-1)}{(x-1)(x+2)} = \frac{x+1}{x+2}; \qquad x \neq 1$$

Information about $x \neq 2$ we didn't lose, it's still in our final expression.

Equality, which is true for some particular values of the variable(s) included in it, is called equation. To solve the equation, we need to find all possible values of variable(s) using the identical transformations, or non-identical transformations, but in this case, we have to remember about anything which can arise from it.

Inequalities.

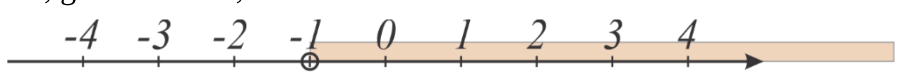
There is another type of problems, when we need ti find all possible values of varible which are greater (or smalle) than a particular number. In more sofisticated case, for which values of variable, one expression is greater (smaller) than another expression, for example:

$$x + 3 > 2x - 5$$

The simplest inequality is

$$x > a, \qquad x < a, \qquad where\ x\ is\ variable\ and\ a\ is\ a\ number.$$

$x > -1$,   the solution is all x, greater than 1,



Solution can be shown graphically are as $x\epsilon(-1, +\infty)$, or can be leaved as it is, it’s already a solution (similarly as a solution of an equation $x = 2$.)

We can add any number to both part of the inequality, the sign (< or >) will not change:

$x > -1$

$x + 2 > -1 + 2 \Rightarrow x + 2 > 1$

$y - 3 < 5$

$y - 3 + 3 < 5 + 3$

$y < 8, \qquad\qquad y \in (-\infty, 8)$

1. $x + 3\ > -5$

Now let’s try to multiply or divide both part of the inequality by the positive number.

If $x > 3$, then $2x$ will be grater then 6.

$x > 3\,,$ $\qquad 2x > 6$

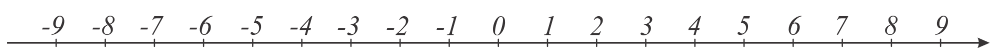
If $x > 3$ what can we tell about $-x$?

$-x \qquad 3 \cdot (-1)$

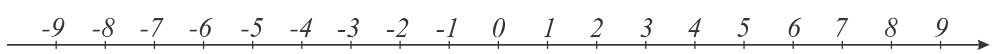
2. $x + 3 \ > \ 5x - 5$
3. $4\mathrm{x} - 3 \neq 0$
4. $3(\mathrm{x} - 1) < 5\mathrm{x} + 9$
5. $2x - 1 > -x + 3$
6. $|x| > 8$

7. Show on the number line points that are satisfying the following inequalities:

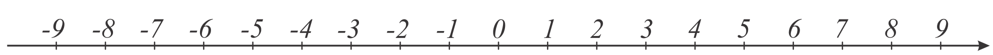
a) $|x| < 4$



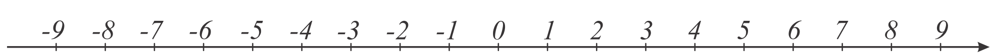
b) $|x| > 3$



c) $\left|x - \frac{1}{2}\right| > 3$



d) $\left|x - \frac{1}{2}\right| < 8$



8. $M = \{x|\ x > 5\},\ K = \{x|x < 20\}$

$M \cap K =$

$M \cup K =$

9. $M = \{x|\ x \leq 5\},\ K = \{x|x \geq 20\}$

$M \cap K =$

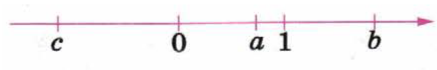
$M \cup K =$

10. Points a, 0, and b are marked on the number line below:

Which of the following expressions is true?

1) $a + b > 0$ or $a + b < 0$
2) $a - b > 0$ or $a - b < 0$
3) $ab > 0$ or $ab < 0$
4) $\frac{b}{a} > 1$ or $\frac{b}{a} < 1$

11. Points a, b, c, 0, and 1 are marked on the number line below:



Which of the following expressions is true?

1) $ab < b \text{ or } ab > b$
2) $abc < a \text{ or } abc > a$
3) $-ac < c \text{ or} - ac > c$

Pyphagorian theorem.

4 identical right triangles are arranged as shown on the picture. He area of the big square is $S = (a + b) \cdot (a + b) = (a + b)^2$, the are of the small square is $s = c^2$. The area of 4 triangles is $4 \cdot \frac{1}{2}ab = 2ab$. But also cab be represented as $S - s = 2ab$

$2ab = (a + b) \cdot (a + b) - c^2 = a^2 + 2ab + b^2 - c^2$

$\Rightarrow \qquad a^2 + b^2 = c^2$