1. Prove that the value of the following expressions is a rational number.
   Example:
   $(\sqrt{3}-1)(\sqrt{3}+1)=\sqrt{3}\cdot\sqrt{3}+\sqrt{3}\cdot 1-1\cdot\sqrt{3}-1=\sqrt{3}\cdot\sqrt{3}-1=(\sqrt{3})^2-1=3-1=2$

   a. $(\sqrt{2}-1)(\sqrt{2}+1)$
   b. $(\sqrt{5}-\sqrt{3})(\sqrt{5}+\sqrt{3})$
   c. $(\sqrt{2}+1)^2+(\sqrt{2}-1)^2$
   d. $(\sqrt{7}-1)^2+(\sqrt{7}+1)^2$
   e. $(\sqrt{7}-2)^2+4\sqrt{7}$

2. Factories polynomials:
   Example: $x^2-5=(x-\sqrt{5})(x+\sqrt{5})$
   $a.\ 4x^2+4\sqrt{5}x+5;$ $b.\ x^2-3;$

3. There are 250 g of cherry jam which has 30 % sugar in it and 300 g of cherry jam with 50 % of sugar in it. Two portions of the confiture were combine together. What is the percentage of sugar in the final product?
4. a. Check the following equalities:

   $\frac{1}{2}-\frac{1}{3}=\frac{1}{2\cdot 3};$ $\frac{1}{3}-\frac{1}{4}=\frac{1}{3\cdot 4};$ $\frac{1}{4}-\frac{1}{5}=\frac{1}{4\cdot 5};$ $\frac{1}{5}-\frac{1}{6}=\frac{1}{5\cdot 6};$

   b. Continue the chain of similar equalities. Write the algebraic expression for the pattern.
   c. Use the previous conclusion to simplify the following expressions:

   $\frac{1}{1\cdot 2}+\frac{1}{2\cdot 3}+\frac{1}{3\cdot 4}+\cdots+\frac{1}{n(n+1)};$

   $\frac{1}{x(x+1)}+\frac{1}{(x+1)(x+2)}+\frac{1}{(x+2)(x+3)}+\cdots+\frac{1}{(x+99)(x+100)};$

5. Without using calculator compare:

   $3\ \ldots\ \sqrt{11}$ $11\ \ldots\ \sqrt{110}$ $22\ \ldots\ \sqrt{484}$
   $5\ \ldots\ \sqrt{20}$ $17\ \ldots\ \sqrt{299}$ $35\ \ldots\ \sqrt{1215}$

6. Each phone number has 10 digits: (NXX) NXX-XXXX. How many different phone numbers may be created in the US? The area code (first tree digits of the number) and the number itself can't start with 0 or 1. (N can be 2-9 and X can be 0-9)